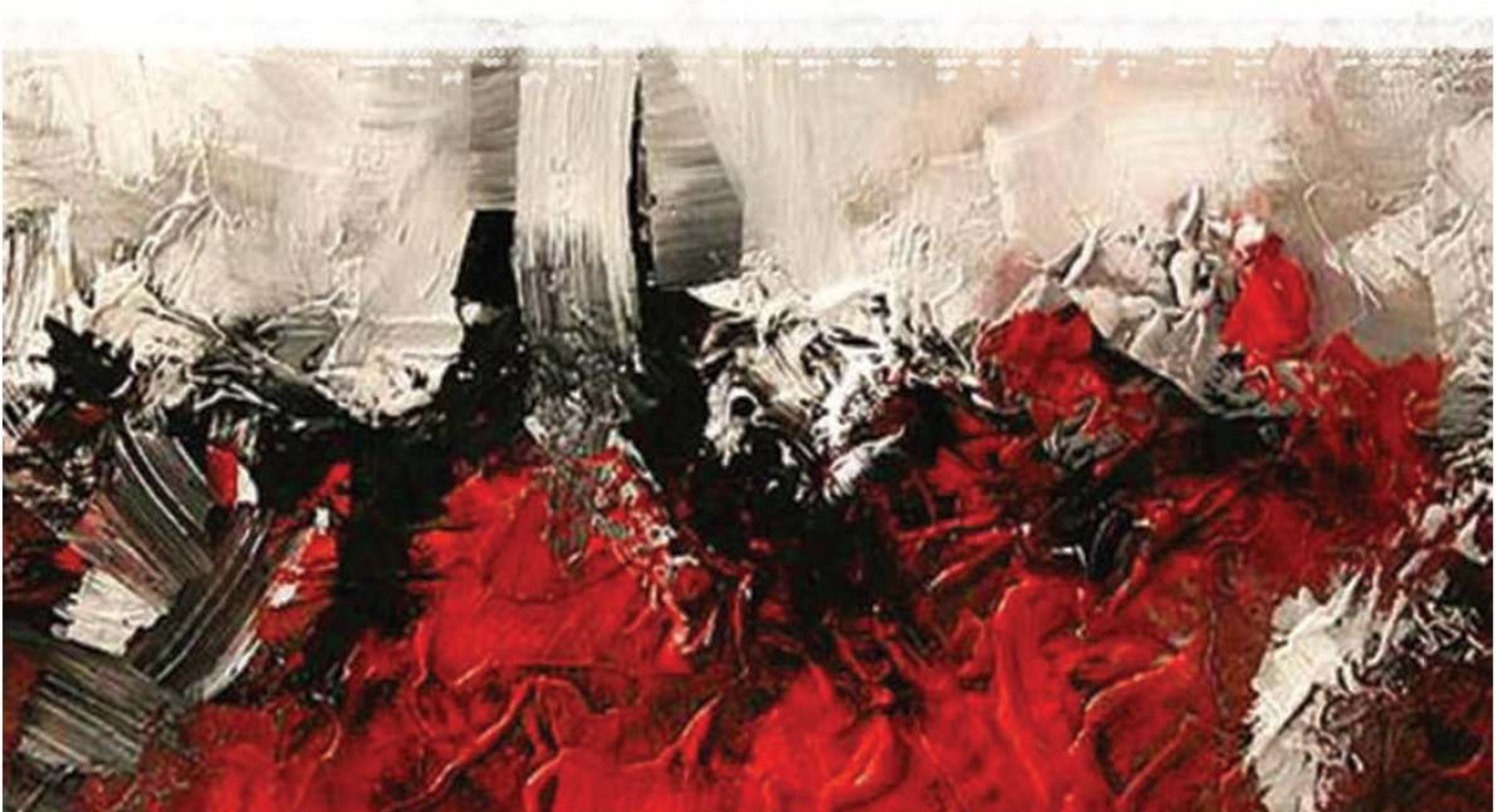
ALGEBRA 2

Inequalities & Quadratic Equations



MATH SCRIBER

RAJ WIJESINGHE

If you keep on going
And never stop,
You can keep on going,
You can make it to the top.
Life is full of mountains,
Some are big and some are small,
But if you don't give up
You can overcome them all.
So keep on going
Try not to stop,
When you keep on going
You can make it to the top.

INEQUALITIES

Solve the following inequalities

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[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

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INEQUALITIES

[redacted]

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INEQUALITIES

[redacted]

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[redacted]

[redacted]

[redacted]

[redacted]

24. $-2x^2 + 3x + 2 < 0$

[redacted]

[redacted]

[redacted]

[redacted]

INEQUALITIES

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INEQUALITIES

[redacted]

[redacted]

59. Sketch the graph of $y = |2\sin x - 1|$ in the range $0 \leq x \leq 2\pi$.

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INEQUALITIES

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INEQUALITIES

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[redacted]

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[redacted]

INEQUALITIES

[redacted]

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[redacted]

[redacted]

83. [redacted]

(i) Obtain the intersection point of the curves

[redacted]

[redacted]

[redacted]

86. (i) Sketch the graphs of $y_1 = |3x+1|$ and $y_2 = |2-x|$.

[redacted]

[redacted]

[redacted]

[redacted]

89. [redacted]

Using the graph obtain the answer of $|x^2 - 9| = x - 3$.

[redacted]

90. Function f is defined as $f : x \rightarrow \frac{x+4}{x-4}$ $(x \neq 4)$.

(i) Show that f (4 + k) + f (4 - k) = 2 if k is a real number.

(ii) Take the value of q as f (q) = 2q – 3.

91. (a) [redacted]

(ii) Sketch a rough sketch of the curves $y = x^2 - 4x + 3$

[redacted]

92. (a) [redacted]

(ii) Sketch the rough sketch of the curves given by

[redacted]

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[redacted]

Find the range of x for $(x^2 - x - 2)(x^2 + x + 1)$ to be positive.

[redacted]

99. Solve the inequality $\left|\frac{2x-1}{x+1}-3\right|<1$.

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INEQUALITIES

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[redacted]

[redacted]

111. [redacted].

(1) [redacted] (3) $\alpha^2 + \beta^2$

([redacted] (5) $\alpha^3 + \beta^3$ [redacted]

112. [redacted] (6) $\alpha^4 + \beta^4$

[redacted]

[redacted]

[redacted]

115. [redacted].

(1) [redacted]

[redacted] $\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}$ [redacted]

[redacted]

121. [redacted] .

(1) [redacted] (3) $(\alpha + \frac{1}{\alpha})(\beta + \frac{1}{\beta})$

[redacted]

[redacted]

[redacted]

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[redacted]

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[redacted]

[redacted]

[redacted]

[redacted]

138. [redacted] $c\,(a-b)^2 = (b-c)(ac-b^2)$.

139. [redacted]

[redacted]

145. $x^2 + ax + b = 0$ [redacted]

146. [redacted] $4x^2 - 1 = 0$

[redacted]

149. $x^2 - cx + d = 0, \quad x^2 - ax + b = 0$ [redacted]

common

[redacted]

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166. (1) If $a > 0$ and $b^2 - 4ac < 0$, Show that the quadratic function

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176. [redacted]

[redacted] Find the values of λ where the equation $p(x) = 0$ has two

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196. [redacted]

(a) Show that f(x) does not lie between $-7-4\sqrt{3}$ and $-7+4\sqrt{3}$, for any real value of x.

(b) Express f(x) in the form $A+\frac{B}{x-4}+\frac{C}{x-3}$, where A, B and C are constant.

Hence, or otherwise, find the maximum and minimum of f.

(c) Find the equations of the vertical and horizontal asymptotoes of f(x).

(d) Sketch the graph of f(x).

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[redacted]

[redacted]

201. [redacted]

(b) A number is divisible by 11. Let α be the number obtained by adding the digits in the odd positions of the number and b be the number obtained by adding the digits in the even positions of the number. Show that a - b is divisible by 11.

202. (a) If n is an odd positive integer and $n > 2$ show that if $x^n + 2$ is divided by $x^2 - 1$ the remainder will be $x + 2$.

(b) Show that if a number is divisible by 9 then the sum of the digits of the number is also divisible by 9.

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208. If α, β are the roots of this equation when $\lambda = 5$, find the equation whose roots are $(\alpha + 2)^{-2}$ and $(\beta + 2)^{-2}$

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214. Find the condition to be satisfied by a, b, c for one root of the equation $ax^2 + bx + c = 0$ to be the square of the other root.

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221. [redacted] the roots of the equation $px^2 + qx + r = 0$. find the equation whose

[redacted]

[redacted]

224. [redacted] (ii) $\frac{3-\alpha}{2+\beta}+\frac{3-\beta}{2+\alpha}$

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[redacted]

228. If one root of equation $ax^2 + bx + c = 0$ is the square of the other root, prove without solving the equation $c\,(a - b)^3 = a\,(c - b)^3$

[redacted]

[redacted]

[redacted]

234. If the roots of the equation $x^2 + ax + b = 0$ are α, β express the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ in terms of a and b.
find the equation whose roots are $(\alpha - 1)^2$ and $(\beta - 1)^2$

235. If the roots of the equation $ax^2 + bx + c = 0$ are α, β and the roots of the equation $apx^2 + bqx + cr = 0$ are α, γ prove that:
$ac(r - p)^2 = (p - q)(q - r)$
Find the values of α, β, γ as rational functions of coefficients.

236. [redacted]
$\alpha_1\alpha_2 + \beta_1\beta_2$ and $\alpha_1\beta_2 + \beta_1\alpha_2$ is $x^2 - kb^2x + 2k^2c(b^2 - 2c) = 0$
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248. If α, β are the roots of the equation $x^2 - 4x + 1 = 0$, prove that $\alpha^3 + \beta^3 = 52$ and deduce that $\alpha^6 + \beta^6 = 2702$

[redacted]

250. If $S = px^2 + 2qx + r$ and $S_1 = x^2 + c$, find in terms of λ the condition

that should be satisfied for the equation $S + \lambda S_1 = 0$ to have equal roots.

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[redacted]

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260. [redacted]

[redacted] roots are $\frac{\alpha}{2\beta+3\alpha}, \frac{\beta}{2\alpha+3\beta}$.

261. [redacted]

[redacted].

[redacted]

[redacted]

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[redacted]

266. [redacted] .

[redacted] .

show that $\lambda + \frac{1}{\lambda} = \left(\frac{q^2-2pr}{pr}\right)$. Hence, if α_1, β_1 are roots of $a_1x^2 + b_1x + c_1 = 0$ and α_2, β_2 are roots of $a_2x^2 + b_2x + c_2 = 0$, show that $\lambda_1 + \frac{1}{\lambda_1} = \lambda_2 + \frac{1}{\lambda_2}$ where $\lambda_1 = \frac{\alpha_1}{\beta_1}$ and $\lambda_2 = \frac{\alpha_2}{\beta_2}$. When it is given that $\alpha_1 b_2^2 c_1 = \alpha_2 b_1^2 c_2$, show that $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$ or $\frac{\alpha_1}{\beta_2} = \frac{\beta_1}{\alpha_2}$.

267. α, β are roots of quadratic equation $f(x)ax^2 + bx + c = 0$.

(i) Find the equation whose roots are $\alpha - 1$, $\beta - 1$.

(ii) Find the requirement that both α and β are greater than 1.

(iii)Guadratic equations $ax^2 + bx + c = 0$ and $ax^2 + cx + b = 0$ are not identical. If they have common roots, show that $a + b + c = 0$. Find the equation that satisfies the remaining roots of two equations.

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289. [redacted]

(ii) Show that $(p+q-r)x^2-5(p+q)x+p+q+r=0$ has real roots.

[redacted]

290. a, b, c are real in $(a-b-c)x^2+2(a-b)x+a-b+c=0$. show that roots of the equation are real distinct rational.

When $c=0$, deduce that roots are real distinct real real coincident.

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300. [redacted].

[redacted]w that $ax^2 - 4bx + c = 0$ is the equation whose

[redacted]

[redacted]

(iii) Find the value of a and value range of a that makes the roots real and distinct in the equation $f(x) = (x^2 + 2)(a - 1) = 0$. Find a when roots are real coincident.

[redacted]

303. [redacted]

[redacted] If the function equals to two x values with difference of 4 between then, find two values of k. When $k = \frac{1}{2}$, find two values of x and verify that their difference is 4

304. (i) α, β are roots of the quadratic equation $ax^2 + bx + c = 0$. Find the equation whose roots are $\alpha - 1,\ \beta - 1$ .Get the requirement for both α and β to be higher than 1 .

(ii) [redacted]

[redacted]

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309. Show that $a(x-b)(x-c)+b(x-c)(x-a)+c(x-a)(x-b)=0$

[redacted]

(iv) [redacted].

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[redacted]

[redacted]

[redacted]

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321. [redacted]

$$b(\lambda^2+1)=\lambda(a^2-2b).$$

[redacted]

[redacted]

322. [redacted],

[redacted].

(ii) Find values of a where roots are real coincident .

(iii) Find the value range of a where two roots are (-).

[redacted]

[redacted]

[redacted]

[redacted]

329. α, β are roots of $ax^2 + ax + c = 0$. If $\beta = \alpha^2$ show that $ca^2 + a(c-3) + c^2 = 0$.

Show that $(4c - 6a + 9a^2)x^2 - (2 - 4c + 3a)x + c = 0$ is the equation whose roots are $\frac{\alpha}{2\beta+3\alpha}$ and $\frac{\beta}{2\alpha+3\beta}$.

If $0 < p < r$, show that the function $\frac{x-p}{x^2-2x+p}$ can take all real values for all real values of x .

When $p = \frac{3}{4}$, explain your answers using a graph.

330. [redacted] .

(ii) Get the statements for the sum and product of roots of the quadratic equation in terms of its coefficients . If α and β are roots of $x^2 + px + 1 = 0$, find the quadratic equation whose roots are $\alpha + \lambda$ and $\beta + \lambda$. λ is a constant. Further, if γ, δ are roots of $x^2 + qx + 1 = 0$, prove that $(\alpha + \gamma)(\beta + \gamma)(\alpha - \delta)(\beta - \delta) = q^2 - p^2$.

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[redacted]

[redacted]

[redacted]

[redacted]

336. [redacted]

b, c are -4 and 42 respectively.

[redacted]

338. (i) If $0 < \lambda < 4$, show that the equation $\lambda x(1 - x) + 1 = 0$ does not have real roots.

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350. If $f(x) = x^4 - 4x^3 + 9x^2 - 10x + 7$, then show that the constants p, q, r can be found such that $f(x) = (x^2 + px + q)^2(x^2 + px + q) + r$. Show that $f(x)$ is positive for all real values of x.

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[redacted]

[redacted]

[redacted]

355. [redacted].

Show that $(\alpha-\gamma)(\beta-\gamma)(\alpha-\delta)(\beta-\delta)=(\gamma^2+p\gamma+1)(\delta^2+p\delta+1)$ and deduce that $(\alpha-\gamma)(\beta-\gamma)(\alpha-\delta)(\beta-\delta)=\left(p-\frac{1}{p}\right)^2$.

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[redacted]

365. [redacted]

i) [redacted] [redacted] [redacted]

[redacted]

[redacted]

367. [redacted],

[redacted]

which the roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

[redacted]

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[redacted]

[redacted]

379. If α and β are the roots of $ax^2 + bx + c = 0$ and if γ and δ are the roots of $a'x^2 + b'\mathrm{x} + c' = 0$, prove that the equation with the roots $\frac{\alpha}{\gamma} + \frac{\beta}{\delta}, \frac{\alpha}{\delta} + \frac{\beta}{\gamma}$ is $a^2c'^2x^2 - abb'c'x + a'b^2c' + ab'^2c - 4aa'cc' = 0$.

[redacted]

[redacted]

[redacted].

[redacted]

384. If α and β are the roots of $f_1(x) = 0$ and $\alpha'$and β' are the roots of $f_2(x) = 0$, show that $f_1(\alpha')f_1(\beta') = f_2(\beta) = (q - q')^2 + (p - p')(pq' - p'q)$.

[redacted]

386. Find the solutions of $4x^4 - 7x^3 - 5x^2 + 7x + 4 = 0$.

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[redacted]

[redacted]

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[redacted]

397. Let α and β be the roots of the equation $x^2 + px + 1 = 0$ and let γ and δ be the roots of the equation $x^2 + \frac{1}{p}x+1=0$.

Show that

$(\alpha - \gamma)(\beta - \gamma)(\alpha - \delta)(\beta - \delta) = (\gamma^2 + p\gamma + 1)(\delta^2 + p\delta + 1)$ and deduce that

$$(\alpha - \gamma)(\beta - \gamma)(\alpha - \delta)(\beta - \delta) = \left(p - \frac{1}{p}\right)^2$$

(2001)

398. Let $f(x) = x^2 + 2x + 9$; $x \in R$

(i) If α , β are the roots of $f(x) = 0$, obtain the quadratic equation whose roots are α^2-1 and β^2-1

[redacted] Find the value of a real constant k for which the [redacted]

[redacted]

[redacted]

401. Let $f(x) = x^2 + bx + c$ and $g(x)=x^2 + qx + r$,

where $b,c,q,r \in R$ and $c \neq r$

Let $\alpha.\beta$ be the roots of $g(x) = 0$

Show that $f(\alpha)\, f(\beta)=(c - r)^2-(b - q)(cq - br)$.

Hence, or otherwise, prove that if $f(x) = 0$ and $g(x)= 0$ have a common root,

then b-q, c-r and cq-br are in Geometric Progression.

If $\alpha.\gamma$ are the roots of $f(x) = 0$, show that the quadratic equation whose roots are $\beta.\gamma$ is $x^2 - \frac{(c+r)(q-b)}{(c-r)}x + -\frac{cr(q-b)^2}{(c-r)^2} = 0$

(2005)

[redacted]

[redacted]

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[redacted]

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405. [redacted], whose roots are $\alpha^3\beta^2$ and $\alpha^2\beta^3$ Hence, find the quadratic [redacted]

(2009)

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406. (a) α and β are the roots of the quadratic equation $f(x) \equiv x^2 + px + q = 0$, where p and q are real and $2p^2 + q \neq 0$. If $y(p\text{-}x) = p + x$, substituting for x in $f(x) = 0$ or otherwise, show that $g(y) \equiv (2p^2 + q)y^2 + 2(q - p^2)y + q = 0$, where $y \neq -1$.

Hence, find the roots of the equation $g(y) = 0$ in terms of α and β Express $\left(\frac{\alpha}{2\beta+\alpha}\right)^2 + \left(\frac{\beta}{2\alpha+\beta}\right)^2$ in terms of p and q.

(2010)

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[redacted]

[redacted]

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412. [redacted]

equation, and let $\lambda = \frac{\alpha}{\beta}$ Show that $ac(\lambda+1)^2 = b^2\lambda$.

[redacted]

(2014)

[redacted]

(2015)

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414. (i) Let $f(x) \equiv 2x^4 + \gamma x^3 + \delta x + 1$, where γ and δ are real constants. Given that $f\left(-\frac{1}{2}\right) = 0$ and $f(-2) = 21$, find the two real linear factors of $f(x)$

(ii) Find the two linear expressions $P(x)$ and $Q(x)$ satisfying the equation

$$(x^2 + x + 1)P(x) + (x^2 - 1)Q(x) = 3x, \text{ for all real } x$$

(2015)

[redacted]

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416. Let $a, b, c \in R$ such that $a \neq 0$. and $a+b+c \neq 0$. and let $f(x) = ax^2 + bx + c$. Let α, β be the roots of $f(x) = 0$.

Show that $(\alpha - 1)(\beta - 1) = \frac{1}{a}(a+b+c)$ and that the quadratic equation with $\frac{1}{\alpha - 1}$ and $\frac{1}{\beta - 1}$ as the roots is given by $g(x) = 0$, where $g(x) = (a+b+c)x^2 + (2a+b)x + a$.

Now, let $a > 0$ and $a+b+c > 0$.

Show that the minimum value m_1 of $f(x)$ is given by $m_1 = -\frac{\Delta}{4a}$, where $\Delta = b^2 - 4ac$.

Let m_2 be the minimum value of $g(x)$. Deduce that $(a+b+c)m_2 = am_1$.

Hence, show that $f(x) \geq 0$ for all $x \in R$ if and only if $g(x) \geq 0$ for all $x \in R$.

(2016)

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417. Let $f(x) = 3x^2 + 2ax + b$, where $\alpha, b \in R$.

It is given that the equation $f(x) = 0$ has two real distinct roots. Show that

$a^2 > 3b$. Let α and β be the roots of $f(x) = 0$.

Write down $\alpha + \beta$ in terms of a

and $\alpha\beta$ in terms of b.

Show that $|\alpha - \beta| = \frac{2}{3}\sqrt{a^2 - 3b}$

Show further that the quadratic equation with $|\alpha + \beta|$ and $|\alpha - \beta|$ as its roots is given by

$9x^2 - 6(|\alpha| + \sqrt{(a^2 - 3b)}\ \ x + 4\sqrt{a^4 - 3a^2b} = 0$

(2017)

[redacted]

420. Let $a, b, c \in R$ Write down the discriminant of the equation $3x^2 - 2(a+b)x + ab = 0$ in terms of a and b and Hence, show that the roots of this equation are real.

Let α and β be these roots. Write down $\alpha + \beta$ and $\alpha\beta$ in terms of a and b.

Now , let $\beta = \alpha + 2$. Show that $a^2 - ab + b^2 = 9$ and deduce that $|a| \le \sqrt{12}$,

and find b in terms of a.

(2018)

What's Next ?

VECTORS & COPLANAR FORCES

ALGEBRA 2

Inequalities & Quadratic Equations

Maths Simplified !

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