A/L COMBINED MATHS

CALCULUS

LIMITS & DIFFERENTIATION

RAJ WIJESINGHE

Limits

iv. $\lim_{x\to 1}(1-x)\tan\left(\frac{3\pi x}{2}\right)$

v. $\lim_{x\to a}\frac{\cos x-\cos a}{\sqrt{x}-\sqrt{a}}$

vi. $\lim_{x\to\frac{\pi}{3}}\frac{\sqrt{3}-\tan x}{\pi-3x}$

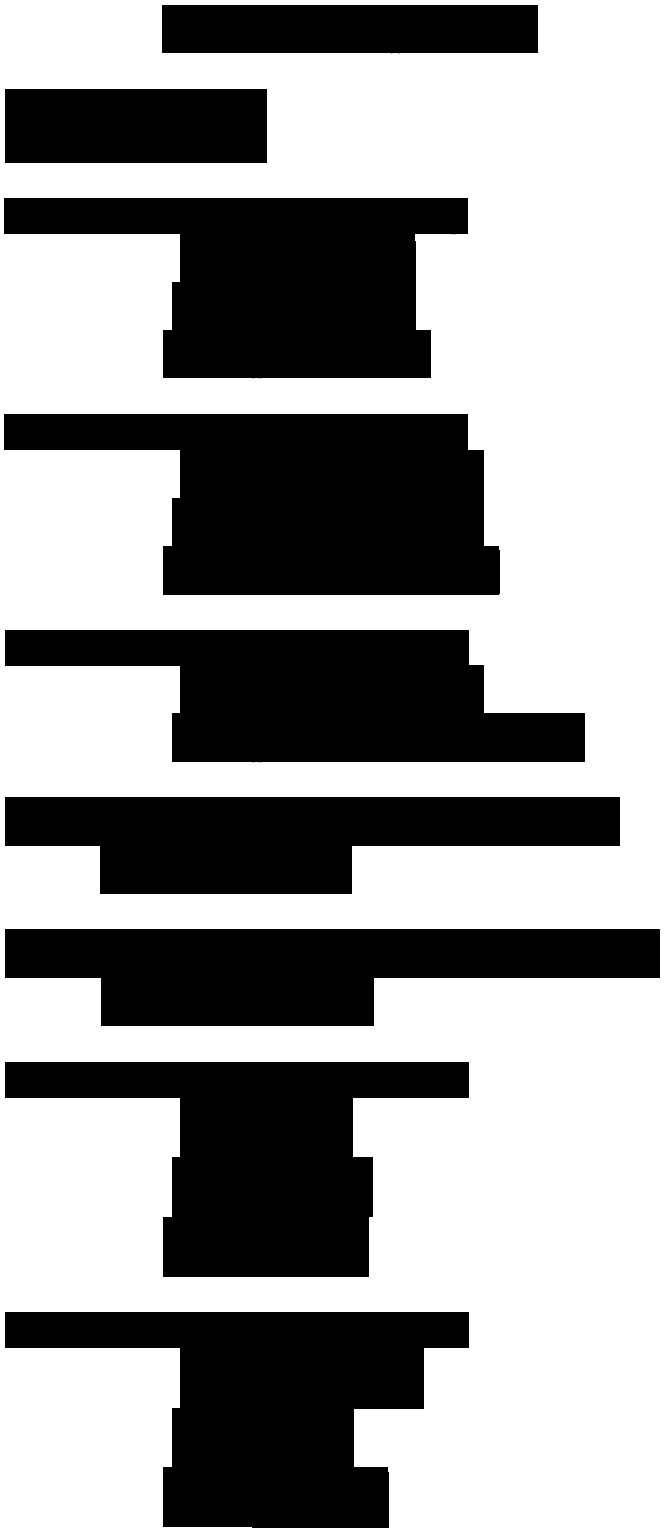
vii. $\lim_{x\to\pi}\frac{\sqrt{2+\cos x}-1}{(\pi-x)^2}$

viii. $\lim_{x\to 0}\frac{4\sin^{-1}x}{3x}$

ix. $\lim_{x\to 0}\frac{\sqrt{4+3x}-\sqrt{4-3x}}{5\sin^{-1}x}$

x. $\lim_{x\to 0}\frac{\left(\sin^{-1}x\right)^2}{2-2\sqrt{1-x^2}}$

Limits



Limits

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26. Show that $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$ and find following.

i. $\lim_{x\to 1}\frac{1-x^{\frac{1}{3}}}{1-x^{\frac{2}{3}}}$

ii. $\lim_{x\to k}\frac{x\sqrt{x}-k\sqrt{k}}{x-k}$

iii. $\lim_{x\to -1}\frac{1+x^{\frac{1}{3}}}{1+x^{\frac{1}{5}}}$

iv. $\lim_{x\to 1}\frac{x^5-1}{x-1}$

v. $\lim_{x\to 1}\frac{x^{\frac{1}{7}}-1}{x-1}$

[redacted]

[redacted]

29. Find the values of following.

i. $\lim_{x\to\infty}\left(\sqrt{3x^2+x+1}-\sqrt{x^2+x}\right)$

ii. $\lim_{n\to\infty}\frac{1^2+2^2+3^3+\cdots+n^2}{n^3}$

iii. $\lim_{x\to\infty}\frac{4x^3+x-1}{x^2+2}$

iv. $\lim_{x\to\infty}\frac{5x^2+x+1}{x^3+x^2-2}$

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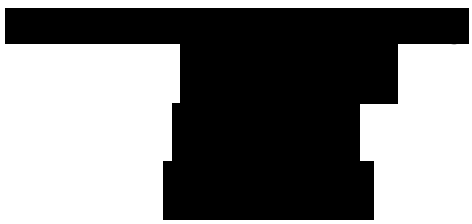
Limits

Limits



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47.

i. Prove by geometrical methods that $sin\, x < x < \tan x$ when $0 < x < \frac{\pi}{2}$. Deduce the limits of $\frac{sinx}{x}$ when the value of x approaches zero across its positive values.

Find the values of

a) $\lim_{x\to 0} \frac{\sin 5x + tan\ 7x}{6x}$.

b) $\lim_{x\to 0} \frac{1+x-\cos x}{\sin x}$.

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Limits

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iv. [redacted]

51.

i. Find the value of $\lim_{x \to 0} \frac{\sqrt{4+x^2}-2}{x^2}$.

ii. Find the value of $\lim_{x \to 7} \frac{\log_e x - \log_e 7}{x-7}$.

iii. Prove that $\lim_{\theta \to \frac{\pi}{4}} \frac{3 sin\theta - 3\cos\theta}{\tan\theta - cot\theta} = \frac{3\sqrt{2}}{2}$.

52.

i. Evaluate $\lim_{x \to a} \frac{\sqrt{5+x^2}-\sqrt{5}}{\sqrt{20+sin^2 x}-\sqrt{20}}$.

ii. If $f(x) = \begin{cases} 1 & x \neq 1 \\ 1998 & x = 1 \end{cases}$, then does $\lim_{x \to 1} f(x)$ exist? Justify your answer.

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54.

i. Evaluate $\lim_{x\to\sqrt{2}} \frac{x^4-4}{x^2+3\sqrt{2x}-8}$

ii. Show that $\lim_{x\to 0} \frac{3^x-1}{\sqrt{1+x}-1} = 2\log 3$.

iii. Find the value of $\lim_{x\to\frac{\pi}{3}} \left(\frac{\sin x-\sqrt{3}\cos x}{x-\frac{\pi}{3}}\right)$.

55.

i. Show that $\lim_{x\to\infty} \frac{4\sqrt{x^2}-\sqrt[3]{x^2+1}}{3^4\sqrt{x^4+1}-\sqrt[4]{x^3+1}} = \frac{4}{3}$.

ii. Show that $\lim_{x\to\infty} \frac{\tan 2x-\sin 2x}{4x^3} = 1$.

iii. Show that $\lim_{x\to\infty} \frac{\sqrt{1+2x}-\sqrt{1-2x}}{\sin^{-1}x} = 2$.

56.

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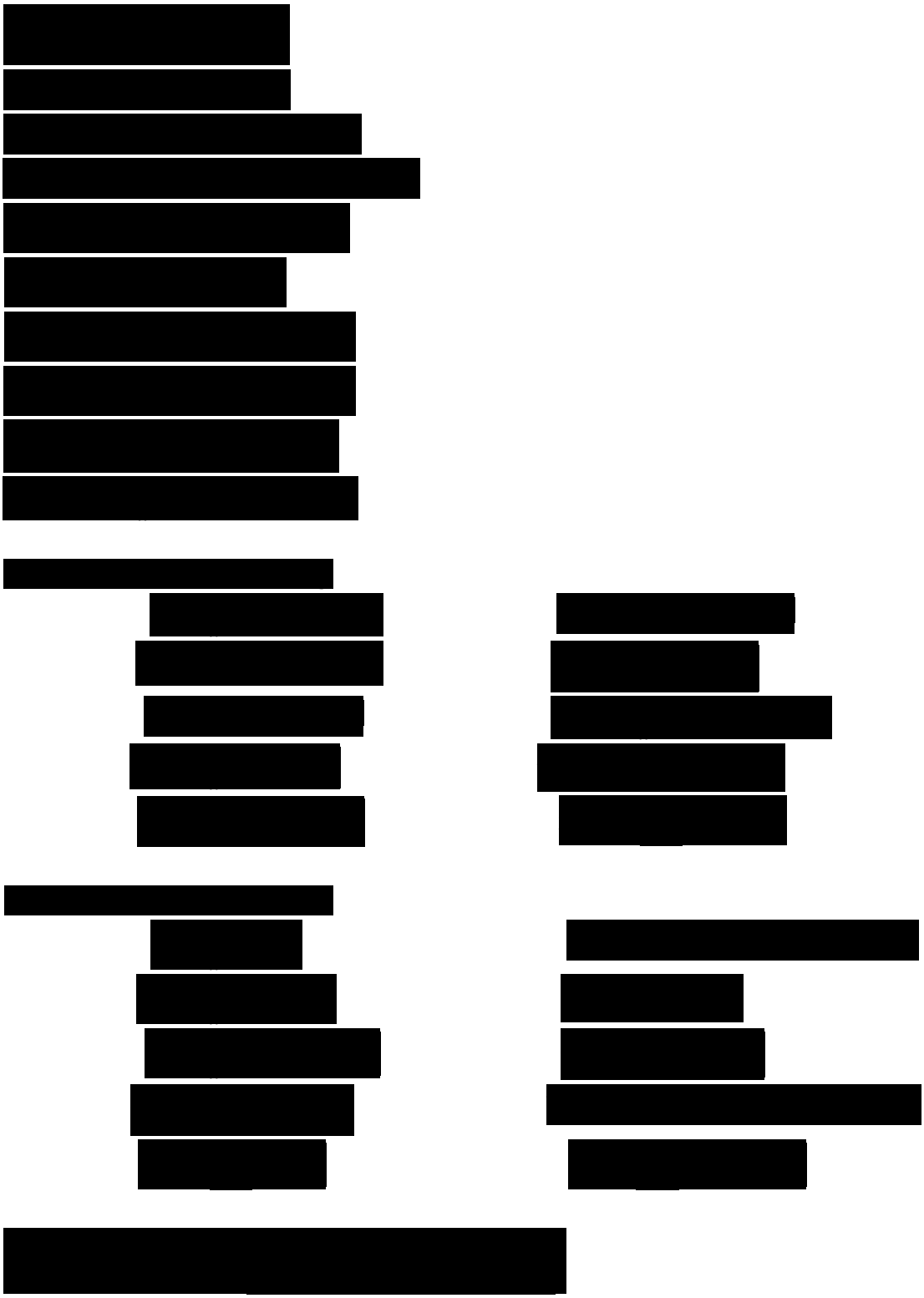

Limits

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Limits

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Differentiation

163. Differentiate $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ with respect to x.

Differentiation

Differentiation

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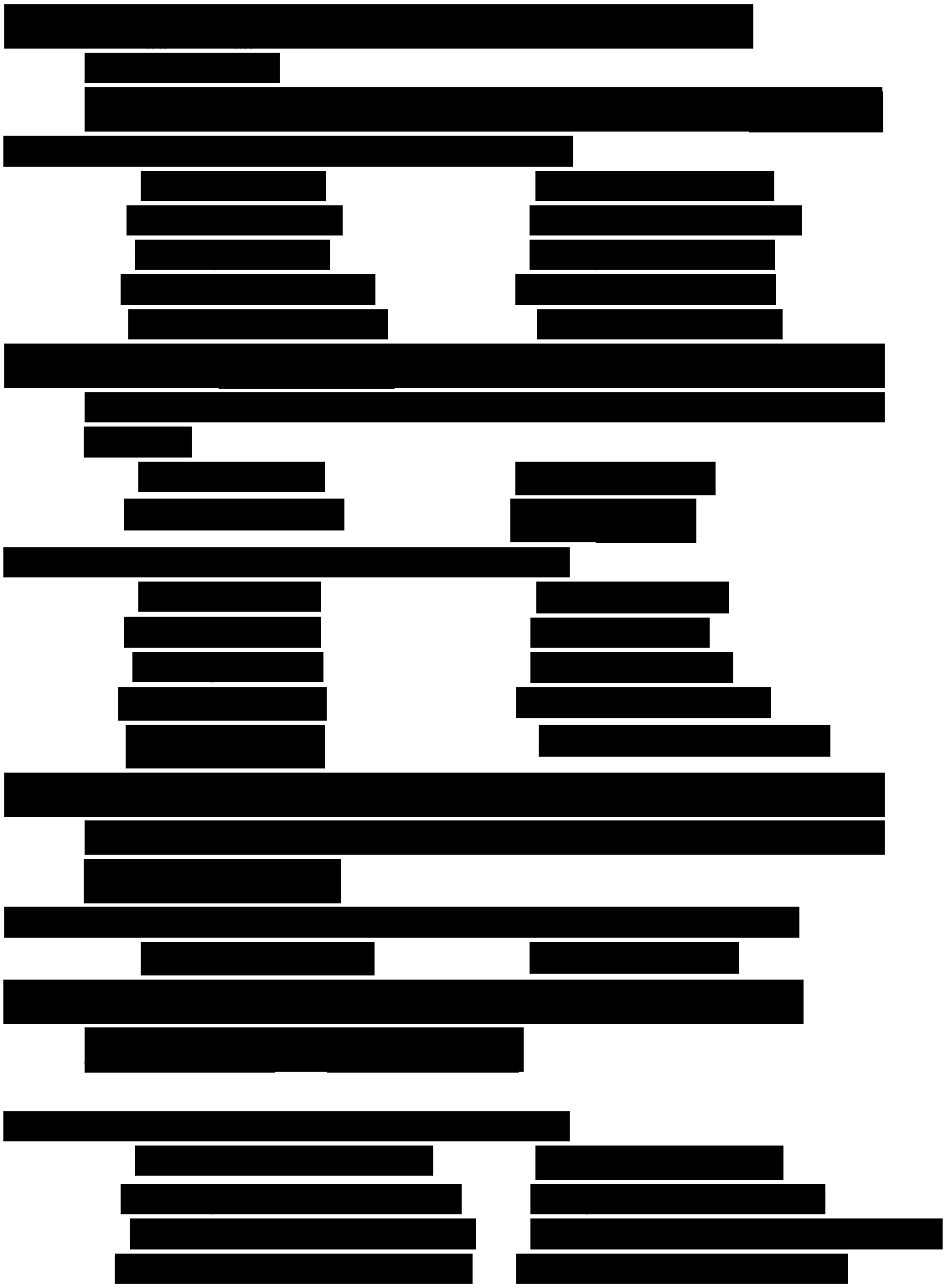
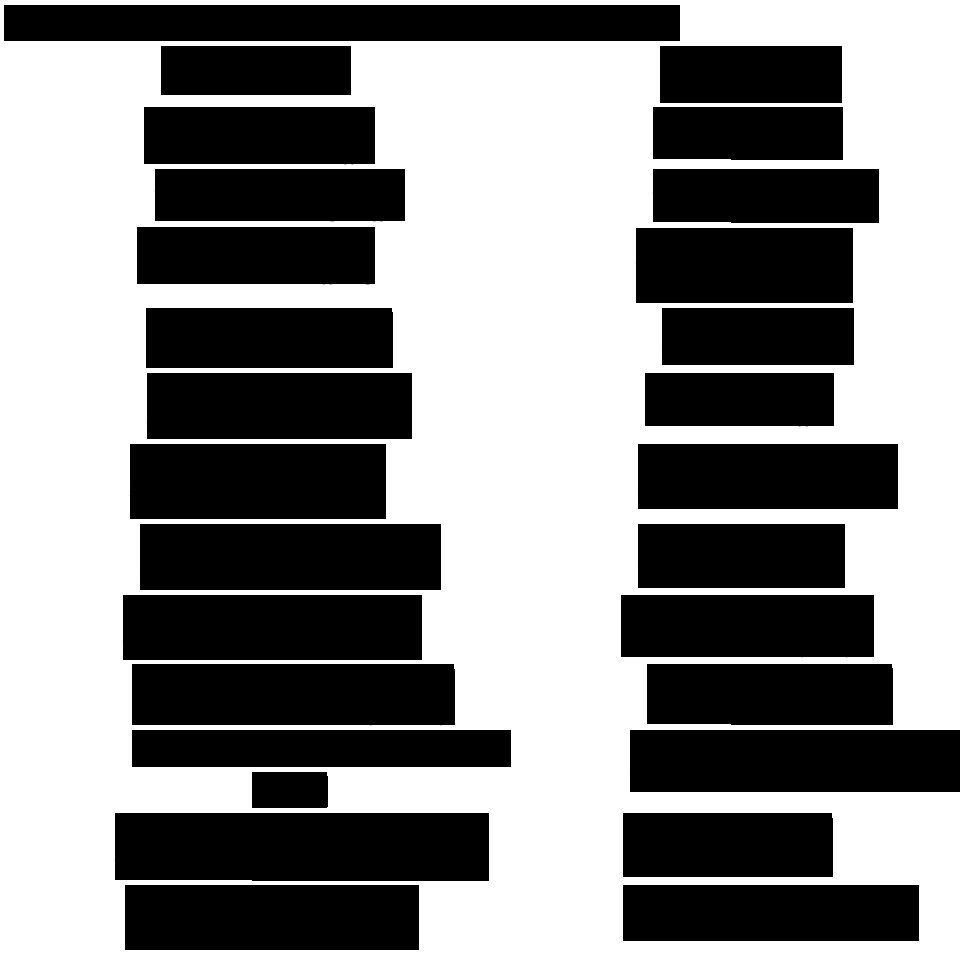
Differentiation



Differentiation



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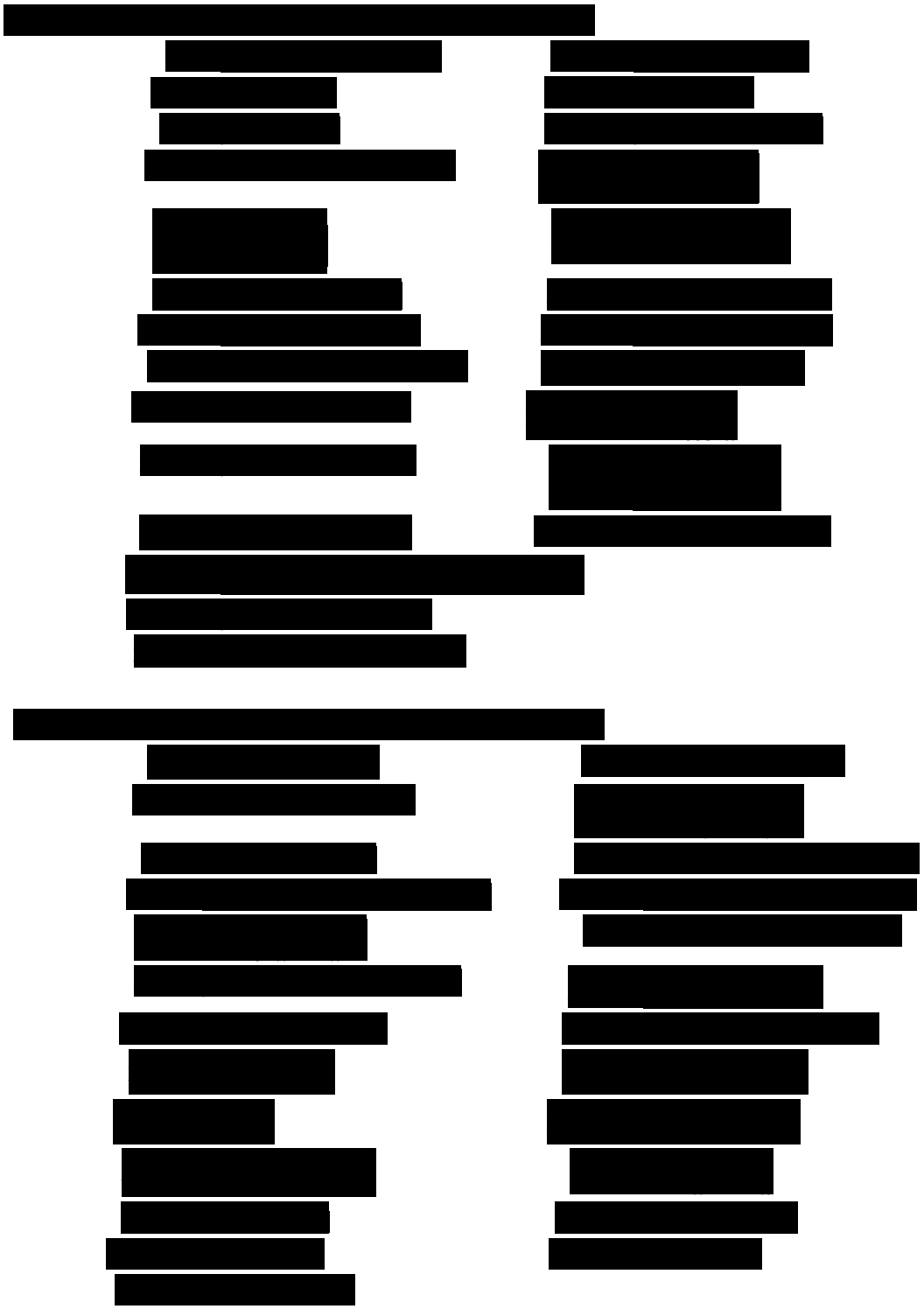
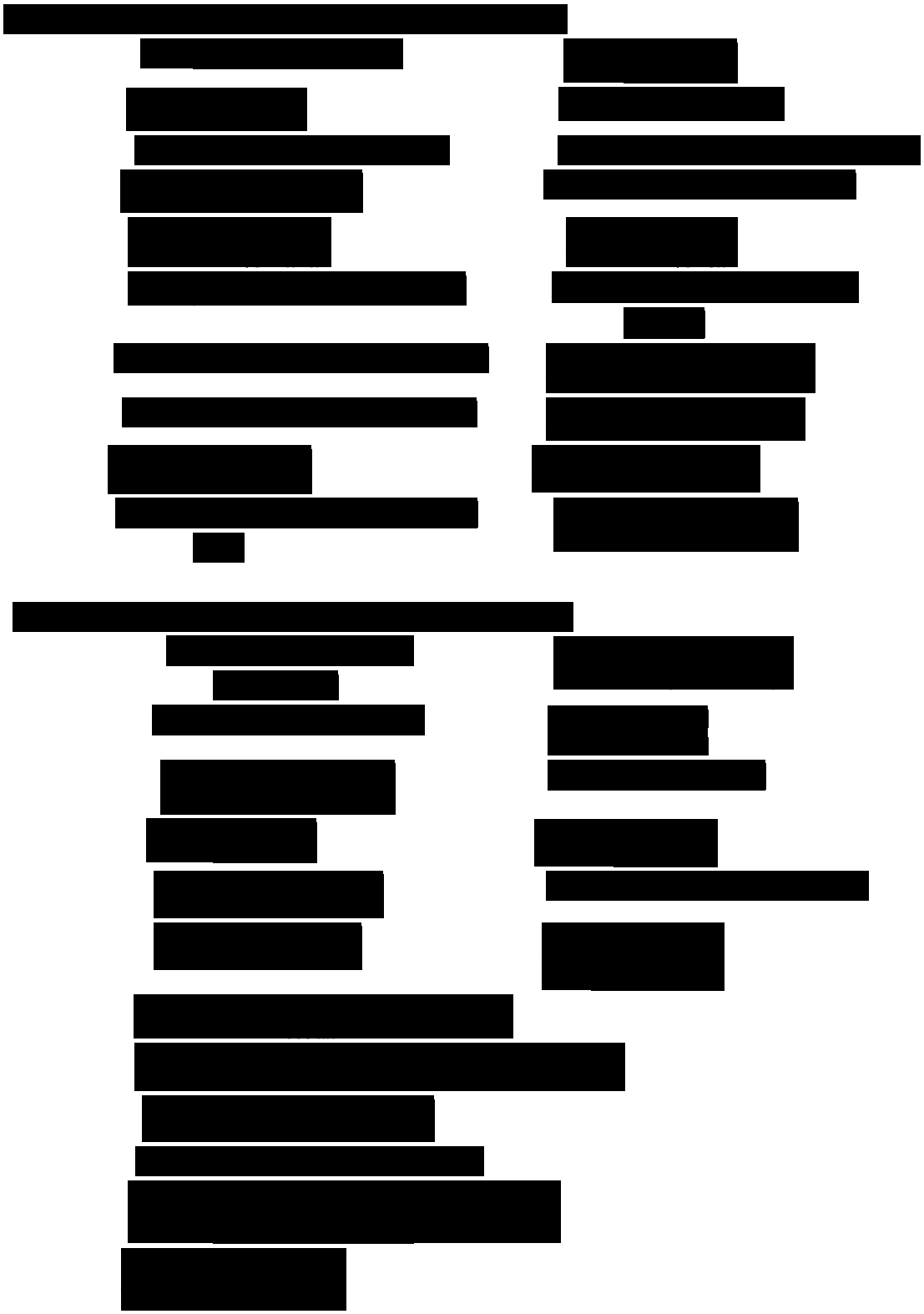


Differentiation



Differentiation

Differentiation

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236. Determine $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for each of the following in terms of x.

i. $x = 2\cos\theta$ and $y = \sin\theta$

241. Let $f(x) = x\,tanx + tanx$, Find the first second and third derivative of $f(x)$, $f'(x), f"(x)$ and $f'''(x)$ *and* also find $f'(0), f"(0)$ and $f'''(0)$.

243. If $y = (Iny + a)x$, Show that $(xy - x^2)\frac{dy}{dx} = y^2$.

246. If $y = \frac{\sec x}{b+tanx}$, Show that $\frac{dy}{dx} = y\tan x - y^2\, secx$.

247. If $y = \frac{k}{k+k\,In\,x+x}$, Show that $x\frac{dy}{dx} + y = y^2\, In\, x$.

248. If $y = x\cos x$, Show that $\frac{d^2y}{dx^2} - \frac{2}{x}\frac{dy}{dx} + \left(\frac{x^2+2}{x^2}\right)y = 0$.

249. If $y = Ae^{-\beta x}\cos(kx + b)$, Show that $\frac{d^2y}{dx^2} - 2\beta\frac{dy}{dx} + (k^2 + \beta^2)\,y = 0$.

250. If $y = e^{-tanx}$, Show that $\frac{d^2y}{dx^2} = (1 + In^2y)(1 - In\,y)^2y$.

255. Let $f(x) = \sin^{-1} x$,

i. Find $f^{(1)}(x), f^{(2)}(x)$ and $f^{(3)}(x)$.

ii. Find $f^{(1)}(0), f^{(2)}(0)$ and $f^{(3)}(0)$ '

256. If $y = \sin^{-1} x + (\sin^{-1} x)^2$, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$.

257. If $y = (a + bx)e^{-\lambda x}$,show that $\frac{d^2y}{dx^2} + 2\lambda\frac{dy}{dx} + \lambda^2 y = 0$ where a, b and λ are constants.

258. If $y = (ax^2 + bx + c)e^{-x}$, show that $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = 0$.

263. If $y = sinx$, find $\frac{d^2y}{dx^2}$ in terms of $t, \frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$. Hence convert the differential equation $\cos x\frac{d^2y}{dx^2} + sinx\frac{dy}{dx} - 4y\cos^3 x = 0$ into $\frac{d^2y}{dx^2} - 4y = 0$.

264. If $y = sinx$, convert the differential equation $\frac{dy}{dx} - 4y\cos^3 x = 0$ into $\frac{d^2y}{dx^2} - 4y = 0$.

265. If $y = \tan x$, find $\frac{d^2y}{dx^2}$ in terms of $t, \frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$. Hence if $2\tan x\frac{dy}{dx} + 4k^2 y\, sec^2 x = 0$.

Show that $\frac{d^2y}{dt^2} + 4k^2 y = 0$.

267. If $y = e^x \cos x$, show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$.

268. If $y = x \sin\frac{1}{x}$, show that $x^4 \frac{d^2y}{dx^2} + y = 0$.

276. The parametric form of a curve is given by $x = at^2, y = 2at$. Find the first Derivative of the curve and hence find the gradients of the tangents drawn to the curve for corresponding values of $t = 1, t = 3$.

277. It is given that $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ Find $\frac{dy}{dx}$ and find the value for $\frac{dy}{dx}$when $\theta = \frac{\pi}{2}$, $\theta = \frac{\pi}{3}$.

291. If $y = (\log_e(x-a))^2$ where $x - a > 0$, find $\frac{dy}{dx}$ and prove that $(x-a)^2\frac{d^2y}{dx^2} + (x-a)\frac{dy}{dx} = 2$. Further more find the value of $\frac{d^2y}{dx^2}$ when $x = 2a$.

294. If $y = 4\sin(\cos x)$ prove that $\frac{d^2y}{dx^2} - 4\cot x\frac{dy}{dx} + y\sin^2 x = 0$.

Differentiation

[redacted]

Differentiation

[redacted]

v. $\tan^2 x$

vi. $\frac{\sin x}{x}$

vii. $\frac{x}{\cos 2x}$

viii. $\frac{\cos(px)}{qx}$ p, q ksh; fõ'

ix. $sec(3x+1)$

x. $cot\sqrt{3x+1}$

323. Differentiate the following funtions with respect to x.

i. $\frac{3x-2}{x+2}$

ii. $\frac{7x+5}{(x-3)^2}$

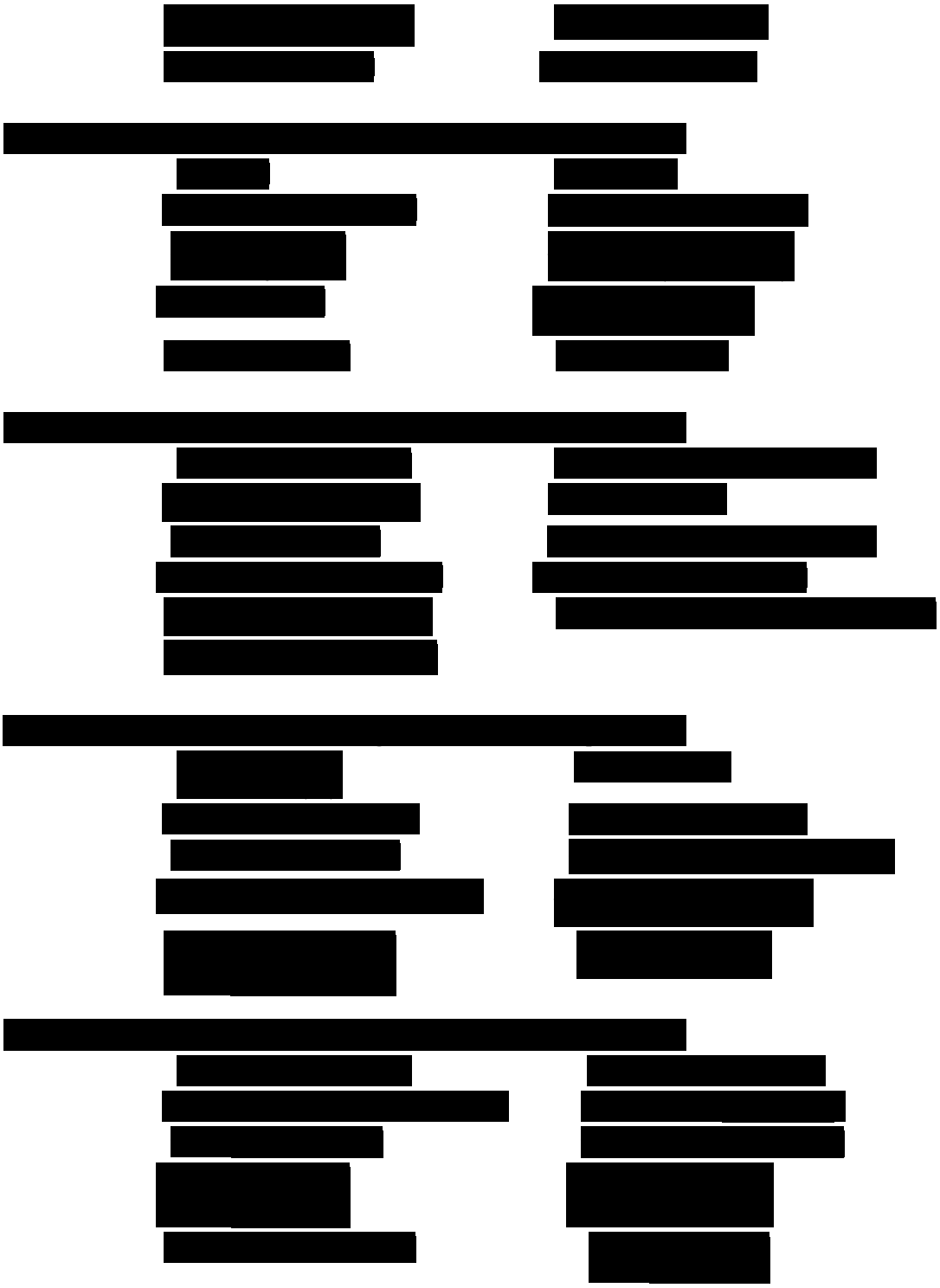
iii. $\frac{(2x+3)^{\frac{1}{2}}}{3x-2}$

iv. $\frac{(3x-7)^3}{(2x+1)^2}$

v. $\left(\frac{\sqrt{x}}{\sqrt{x}+2}\right)^4$

vi. $\frac{ax+b}{px+q}$ p, q are constants

Differentiation



Differentiation

329. Differentiate the following funtions with respect to x.

i. $\frac{\tan^{-1} x^2}{e^{x^2}}$

ii. $\tan^{-1}\left(\frac{a\cos x+b\sin x}{\sqrt{a^2+b^2}}\right)$

iii. $3^x \sin^{-1}(x+1)^2$

iv. $2^x \cos^{-1}(x+2)^2$

v. $\frac{3^x \sin x}{(x+2)^2 \tan^{-1} x}$

vi. $\frac{4\tan^{-1} 2x}{1+4x^2}$

vii. $\sin^{-1}\left(\frac{p\ \sin x+q\cos x}{\sqrt{p^2+q^2}}\right)$ p, q are constants

[redacted]

337. $f(x) = ax^3 + bx^2 + cx + d$, a, b, c, d are constants. Find $f'(x)$ and state $f'(x)$ in the format $f'(x) = A\{(x + B)^2 + C\}$ where A, B, C are constants.

Hence if the following conditions are satisfied, find the values of a,b,c,d and draw a rough sketch of the curve $y=f(x)$

i. For $|x| > 1$, $f'(x) > 0$

[redacted]

341. If $x = 2t + \sin 2t$, $y = \cos 2t$, prove that $\frac{dy}{dx} = -\tan t$.

Furthermore, prove that $\frac{d^2y}{dx^2} = -\frac{1}{4}$

Differentiation

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342. [redacted]

343. [redacted]

[redacted]

[redacted]

[redacted]

[redacted]

347. If u, v are two functions of x, prove that $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$. Furthermore, if u,v,w are functions of x, obtain an expression for $\frac{d}{dx}(uvw)$.

[redacted]

349.

i. Differentiate with respect to x.

a) $\sqrt{x^2+1}\sin 3x$

b) $\cos(\cos\sqrt{x})$

c) $\sqrt{\tan(x^2)}$

ii. Use the first principle to differentiate $\frac{\sin x}{x}$.

iii. If $x^y = 3^{x-y}$, prove that $\frac{dy}{dx} = \frac{x\ln 3 - y}{x\ln 3x}$.

350.

[redacted]

353.

i. Explain the derivative of a function. Obtain the derivative of $\tan x$ using the first principle.

ii.

a) Differentiate $\tan^{-1}\left(\frac{e^x}{\sqrt{1+x^2}}\right)$ with respect to x.

b) Differentiate $\frac{\sin(\cos x)\log|1+x^4|}{x(x^2-1)}$ with respect to x.

[redacted]

354. In the equation $y = f(x)$ the solution in terms of x, y is given by $x = g(y)$. If $\frac{dy}{dx} = \frac{d}{dy}[g(y)] \neq 0$. Prove that $\frac{d}{dx}[f(x)] = \frac{1}{\frac{d}{dy}[g(y)]}$. Differentiate $y = \sin^{-1} x, y = \tan^{-1} x$ with respect to x.
Also find the derivatives of $y = \sin^{-1} x, y = \tan^{-1} x$.
Show that the derivatives of the functions with respect to x,
$2\sin^{-1}(x-1), \frac{2\tan^{-1}\sqrt{x-1}}{(\sqrt{2}-x)}, \sin^{-1} 2\sqrt{(2-x)(x-1)}$ is given by $\frac{1}{\sqrt{(2-x)(x-1)}}$.

355. [redacted]

Differentiation

358.

i. if y is a differentiable function of t and if t is a differentiable function of x show that $\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx}$. Also show that $\frac{d}{dx}(\sin x) = \cos x$ furthermore use the identity $\cos x = \sin\left(\frac{\pi}{2} + x\right)$ and deduce that $\frac{d}{dx}(\cos x) = -\sin x$.

ii. Use logarithm laws to differentiate the funtion $\left(4 + \frac{4}{x}\right)^x$.

[redacted]

Differentiation

[redacted]

[redacted]

368.

[redacted]

ii. If $y = \sin^{-1}\frac{x}{b}$, where $-\frac{\pi}{2} < y < \frac{\pi}{2}, -b < x < b$ find $\frac{dy}{dx}$. Also differentiate $(\alpha)(x^2+1)^{\frac{1}{2}}\sin^2 2x$ and $\sin^2\left(a\sin^{-1}\frac{x}{b}\right), -b < x < b$ with respect to x.

370.

i. If f, g are two differentiable funtions of x, show that $\frac{d}{dx}(fg) = f\frac{dg}{dx} + g\frac{df}{dx}$.

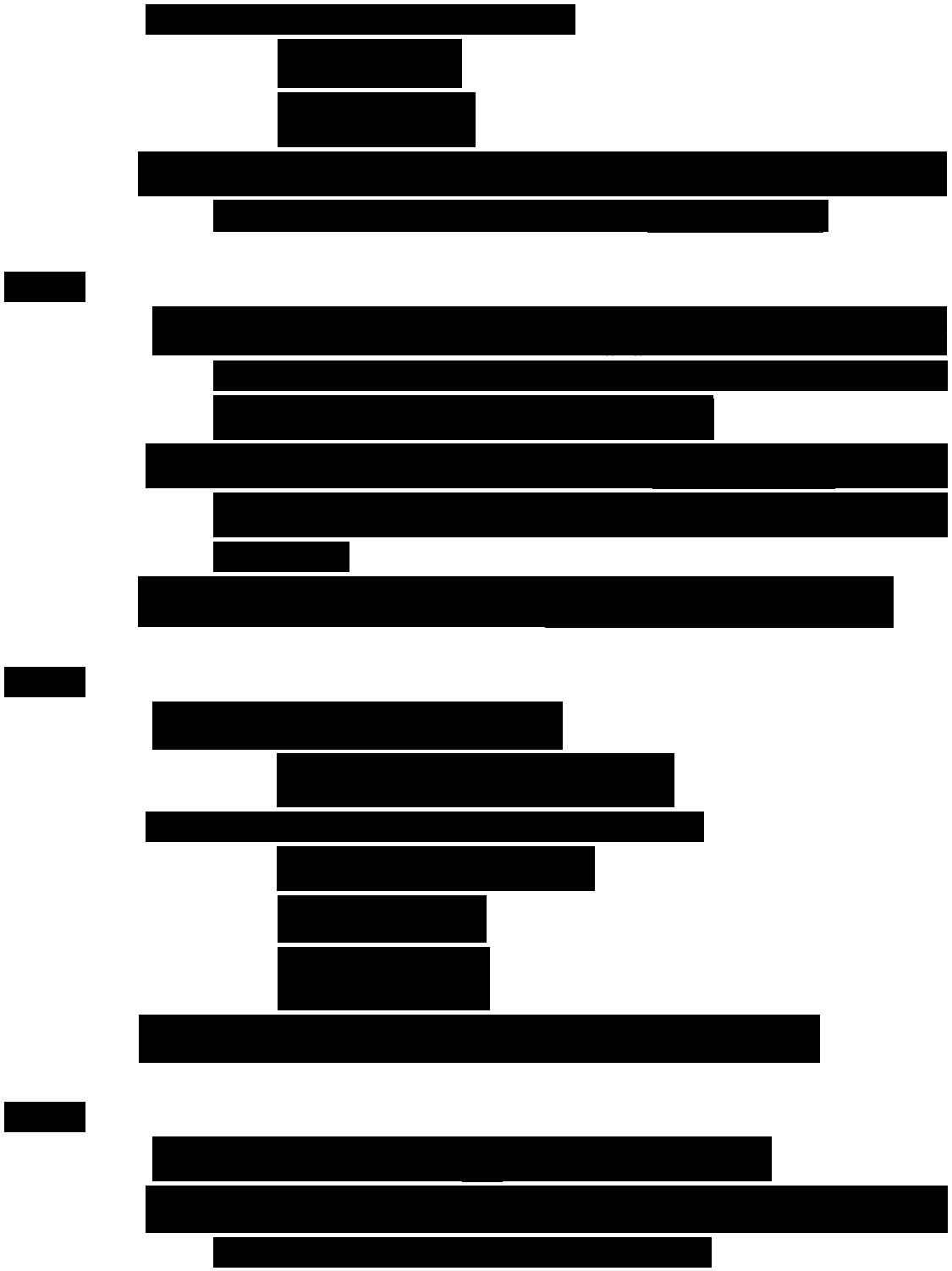
Differentiate the following with respect to x,

a) $e^{x^2}\sin 2x$

b) $\sqrt{x}\sin^{-1}(2x-1)$

c) $\left(\frac{secx+tanx}{secx-tanx}\right)\log_e|secx+tanx|$

Differentiation



Differentiation

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Differentiation

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Differentiation

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386.

389.

i. Find the turning point of $f(x) = x^3(1+x)^{1/2} x > -1$ and draw the sketch of the graph.

ii. Followings are the coordinates of a variable point (x , y) of the curve $x = t(1-t)^2, y = t^2(1-t)$.

In here, t is a variable parameter. Find the equation of the tangent drawn to the curve from the point where $t = \frac{1}{2}$. Show that the curve completely lies on one side of this line.

Differentiation

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[redacted]

[redacted]Find the equation of the tangent drawn to the curve $x^3 + y^3 = 2xy$ at the point of (1, 1). Show that the perpendicular drawn at this point meets the [redacted]

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409. Show that the function $f(x) = \frac{x}{\log_e x}$,

i. Increases in the range of $e < x < \infty$.

ii. Decreases in the range of $0 < x < e$.

410.

Differentiation

[redacted]

417.

426.

i. Using first principles, find the derivative of $f(x) = \sin x$ with respect to x

Deduce the derivative of $g(x) = \cos x$.

Differentiate

a) $\sin[\ln(1 + x^2)]$

b) $\cos(\sin x)$

with respect to x

ii. Let $y = \sin k\theta \; cosec\; \theta$ and $x = cos\theta$, where k is a constant.
Show that,

a) $(1-x^2)\frac{dy}{dx} - xy + k\cos k\theta = 0,$

b) $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + (k^2-1)y = 0.$

iii. The tangent to the curve $y(1+x^2) = 2$ at the point $P\left(3, \frac{1}{5}\right)$ meets the curve again at Q.
Find the coordinates of Q.

2009 A/L

427.

a) Let $y = tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ and $z = tan^{-1}x$. Find $\frac{dy}{dz}$.

b) Let $y = e^{msin^{-1}x}$, Where m is a constant.
Show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$.

[redacted]

431. A curve C is given by the equation $y = 4 - 4x + 3x^2 - x^3$. Find the equation of the tangent drawn to the curve C at the point (1,2) show that this tangent is perpendicular to the tangent drawn to the curve $y^2 = 4x$ at the point (1,2).
The gradient of the tangent drawn to the curve C at the point (1,2)

2012 A/L

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Past Papers

Past Papers

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