MATHSCRIBER

A/L COMBINED MATHS

Calculus II

PRACTICAL APPLICATIONS OF DIFFERENTIATION

RAJ WIJESINGHE

Graphing & Practical Applications

01. Sketch the graph for the equation $y = \frac{3-x^2}{x^2-1}$.

02. Sketch the graph for the equation $y = \frac{x^2-2x+3}{x-2}$.

03. Sketch the graph for the equation $y = x - \sqrt{x}$.

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Graphing & Practical Applications

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31. Find the nature of critical points of the function $f(x) = 5x^3 - 3x^5$and find the nature of the critical points. Also draw a rough sketch of the curve.

32. Find the nature of critical points of the curve $y = \frac{5}{4}x^{4/5}$and find the nature of the critica points. Also draw a rough sketch of the curve.

33. Find the nature of critical points of the function $f(x) = \frac{x^2+1}{x^2-1}$ and find the nature of the critica points. Also draw a rough sketch of the curve $y = f(x)$.

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[redacted] Two poles of heights 5m and 10m are 10m apart from each other. To keep the two poles in equilibrium two strings are connected from the top of both poles to a wedge fixed on the ground between the two poles. In order to minimize [redacted]

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51. Find the cone with the maximum volume which can be contained by a sphere of radius a. if the volume of the cone is V find the maximum value for V.

52. Consider the area of the semi circle bounded by the circle $x^2 + y^2 = 7$ and the axis OX. Find the dimensions of the rectangle with the maximum area which can be contained in this semi circle.

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59. Two squares of side length x are removed from vertices A and D from a rectangular laminar of length 16 m and width 4 m as shown in the diagram. From the vertices B and C, two rectangles with length x and width x+y is also removed from the laminar. By folding along the edges, a box of height x and base ABCD is made. Find the volume of this box. Also find the value of x such that the volume is maximized.

[redacted]

[redacted] A man moves with a velocity $\frac{2\sqrt{3}}{5} ms^{-1}$ along a river bank starting from point A and reaches point B. From point B, he swims across the river of breadth 10 m with a velocity of 2 ms^{-1} and reaches point C at the other bank of the river. The opposite point to C at the other river bank is D where AD is 10m. Obtain an expression for the time taken by the man to move from point A to C. In order to minimize this time, find the length of AB. Also find the minimum time [redacted]

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63. The center is C of a circular lake which has a radius $\sqrt{3}\, km$. A man starting from point P has to reach a point R on the diameter PCR. In order to reach R, the man swims with a velocity $\sqrt{3}\, kmh^{-1}$ starting from point P and arrives to point Q on to the bank of this lake. It is given that the angle $QPR <= \theta$. Thereafter, to reach R, the man walks along the bank of the lake with a velocity 2 kmh^{-1} . The time taken for the man to travel from P to R is T. Obtain an expression for T. In order for T to be maximum or minimum, find the location of Q. In order to maximize the time, find the length of the locus PQR.

64. A,B,C are three cities which are connected by two direct routes, AB and BC such that AB is 15 km, BC = 50 km and $ABC <= \frac{\pi}{2}$. There is a planned project to connect the city A to a point D which is located in route BC. A car can travel with a maximum velocity of 50 kmh^{-1} along DC and with a maximum velocity of 40 kmh^{-1} along the proposed route AD. If point D is located x km from A, find the time $T(x)$ in hours, which a car takes to travel from A to C through D. You may assume that the car travels in its maximum velocities. Examine the sign of $\frac{dT}{dx}$ when x increases from 0 to 50km. Find the most suitable position for D in order for thr car to travel from A to C in the minimum amount of time.

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84. Find the maximum and minimum values of the following functions and sketch the graphs.

$(a)\ \frac{1}{x-1} - \frac{4}{x-2}$ $\qquad (b)\ e^x(x^2 - 3)$

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89. For the function $y = f(x)$ at the point $x = a$, to have ,

    a) a maximum

    b) a minimum

    Find the sufficient requirement states only in the terms of $\frac{dy}{dx}$ At the point $x = a$, show that

    i. The function $y = x^3$ has no turning point,

    ii. The function $y = x^4$ has a turning point.

    Draw a sketch of the graph of $y = x^3$. Obtain the sketch of the graph of $y = x^4$ by using that. State the ideas of the shapes of the graphs of $y = x^{2n+1}$ and $y = x^{2n}$ where n is a positive integer. by the same set of argument.

90.

    i. Draw the sketch of the graph for the function $y = \frac{x}{x^2+1}$.

    ii. A cylindrical container is to be made by using a given uniform raw material. Show that the highest volume for that container can be there when its height is equals to the radius of its base.

91.

[redacted]

What are the maximum and minimum values of y? Considering the intersection of the above curve and the line $y = k$, show that there are no real roots for the equation $(k-1)^2x^2 - 3kx + 2(k-1) = 0$, if $-2(3\sqrt{2}+4) <$ [redacted]

[redacted]

98.

i. Draw the sketch of the graph of $y = x - 1 + \frac{1}{x+1}$. Deduce that the equation $x - 1 + \frac{1}{x+1} = k$ has no real roots when $4 < k < 0$

[redacted]

ii. When t is a parametry, the equation $x = \frac{t^2}{1+t^2}, y = \frac{t}{1+t^2}$ defines a curve. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point t of the curve. By this, find the stationary points and the nature of the curve.

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100. [redacted]

103. Draw the curve given by the following parametric equations.
$x = 2a\cos\theta - a\cos 2\theta$
$y = 2a\, sin\theta - a\, sin2\theta, \ \ (0 \leq \theta \leq \pi)$
Obtain a relationship between x and y, by removing θ from the above equations. How would the curve given by the equation derived without θ differ from the first curve?

104. A curve is defined by the following parametric equations.
$x = 2t^2 - t^4$
$y = 3t^2 + 2t^3, -\infty < t < \infty$
Find the equations of the perpendicular and the tangent drawn to the curve at the point where t is parametric. Find the equations of those specially when

$t = (-1, 0)$ and 1. Draw the parts of the curve in which $t > 0$ and $t < 0$ in seperate diagrams.

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107. A curve is given by $y = \frac{(x-2)^2}{x^2+4}$. Show that,

i. $0 \leq y \leq 2$

ii. when $y \to 1$, it is $x \to \pm\infty$

By this means or others write the coordinates of the turning points of the curve, and then draw the sketch of it.Draw the sketch of the curve C' given by $y = \frac{(x-2)^2}{x^2+4}$ on the same diagram by using the curve C and the symmetrical properties.Find the area of S surrounded by the curve C and the lines $x = 2, y = 1$.By this means or others deduce the area of S', made by the line $x = 2$ and the combined curve of curves C and C' in the region of $0 \leq x \leq 2$. Find the volume of the solid generated by the rotation of S' by the angle of π rad at the axis of $y = 1$.

108.

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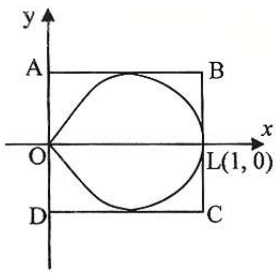
112. A particle moves in a velocity of v ms^{-1} at a time of t s to the positive direction of Ox axis. It is given that $v = 3(4 - 3e^{-2t})$.

   i. Show that $\frac{dv}{dt} = 2(12 - v)$

   ii. Obtain that $\left(\frac{v}{12-v}\right)\frac{dv}{dt} = 2v$ and $\frac{v}{12-v} = \frac{12}{12-v} - 1$, when the particle is at x distance from O. By this means, show that distance travelled by the particle is $\frac{1}{2}[12\log_e 2 - 3]m$, when its velocity increases from 6ms^{-1} to 9ms^{-1}.

113. [redacted]

[redacted]



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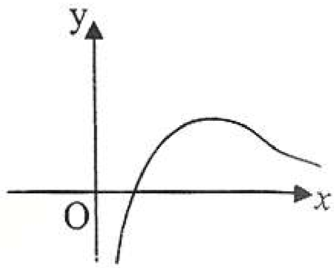
116. The diagram shows the curve $y = \frac{\log_e x}{x^2}, x > 0$

i. Find the point at which the Ox axis is intersected by the curve.

ii. Show that $\frac{dy}{dx} = \frac{1 - 2\log_e x}{x^3}$, and find the coordinates of the maximum point by that means.

iii. Find the values of $\frac{d^2y}{dx^2}$ and $[\frac{d^2y}{dx^2}]_{x=2}$



117.

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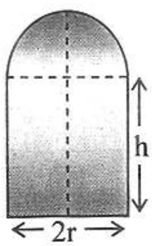

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122.

i. Show that $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 5$, if $y = \sqrt{5x^2 + 3}$.

ii. The paramteric equation of a curve is $x = 3(2\theta - \sin 3\theta), y = 3(1 - \cos 2\theta)$. The tangent drawn to the curve at the point P where $\theta = \frac{\pi}{4}$ and the perpendicular at the same point, meet y axis at L and M respectively. Show that the area of the triangle PLM is $\frac{9}{4}(\pi - 2)^2$.

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124. Find the value of $\lim_{x\to 0} \frac{\cos 3x - \cos x}{\cos 4x - \cos 2x}$. Differentiate the followings with respect to x,

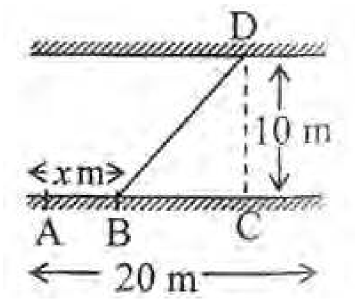
i. $\frac{\sin x}{\sqrt{\cos x}}$ ii. $x^4 \tan 4x$

In an isosceles triangle, two equal sides are such that $AB = AC = a$ and $A = 2\theta$. Show that the radius of the inscribed circle of the triangle is $r = \frac{a \sin\theta \cos\theta}{1+\sin\theta}$. If a is a constant and θ varies $(0 \le \theta \le 90)$, then find that the maximum value for r, records when $\theta = \sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$.

125.

[redacted]

iii. A man moves a distance of x m from A to B at a velocity of $\frac{5}{4} ms^{-1}$. Then he swims from B to D at a velocity of 1 ms^{-1}. C is the exactly opposite point to D. CD = 10 m, AC = 20 m. Show that $t = \sqrt{100 + (20-x)^2} + \frac{4x}{5}$,Where t is the time taken to reach A from D. Find the value of x when t is at its minimum.

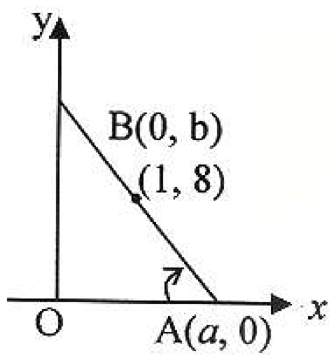


126.

i. Find $\frac{dy}{dx}$ when, $2ye^{3x} + \frac{1}{x^2}\sin 2x = 0$

ii. Find , $\frac{dy}{dx}$, if $x = \frac{1+t}{1-2t}, y = \frac{1+2t}{1-t}$. Find the value of $\frac{dy}{dx}$ when t=0.

[redacted] Differentiate with respect to x, $\log_{10}(k \sec x) + 10^{3x-2}$, where k is [redacted]



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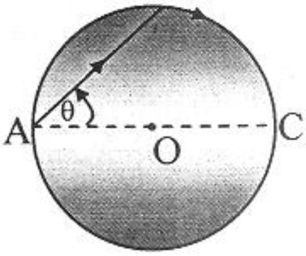

128. Differentiate with respect to x,

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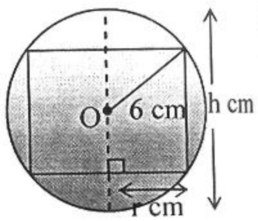

a) Show that $h^2 + 4r^2 = 144$

b) Find the maximum volume of the cylinder.

129.

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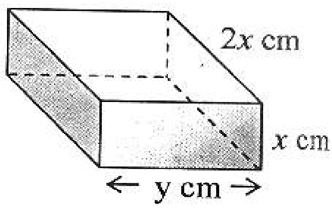

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140.

a) By considering **only the first derivative** find the minimum and maximum values of $\frac{x^3}{x^4+27}$.

Sketch the graph of $y = \frac{x^3}{x^4+27}$.

Hence, find for what values of k, the equation $kx^4x^3 + 27k = 0$, where k is real, has

- a) Two coincident real roots,
- b) Three coincident real roots,
- c) Two distinct real roots,
- d) No real roots.

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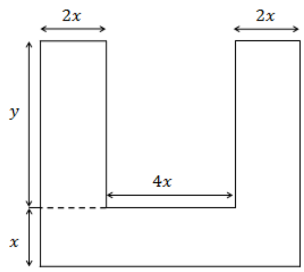

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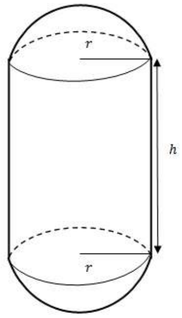

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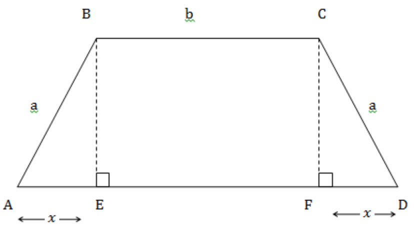
145. In the given figure, ABCD is a trapezium with parallel sides BC and AD. Lengths of its sides, measured in centimetres are given by AB=CD=a, BC=b and AD=b+2x, where $0 < x < a$. BE and CF are the perpendiculars drawn from the vertices B and C, respectively, on to the side AD.



Show that the area S (x) of the trapezium ABCD is given by S(x)=(b+x)$\sqrt{a^2 - x^2}$ in square centimetres. If $a = \sqrt{6}$ and b=4, show further that S(x) is maximum for a certain value of x, and find this value of x and the maximum area of the trapezium.

2015 A/L

146.

a) Let $f(x) = \frac{(x-3)^2}{x^2-1}$ for $x \neq \pm 1$.

Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x), = \frac{2(x-3)(3x-1)}{(x^2-1)^2}$.

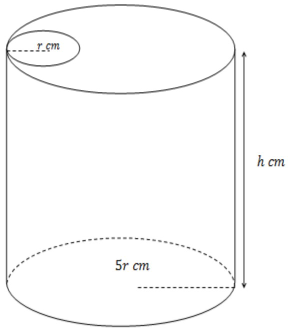
Write down the equations of the asymptotes of $y = f(x)$.

Find the coordinates of the point at which the horizontal asymptote intersects the curve $y = f(x)$

[redacted]

b) A thin metal container, in the shape of a right circular cylinder of radius $5r$ cm and height h cm has a circular lid of radius $5r$ cm with a circular hole of radius r cm. (See the figure.) The volume of the container is given to be $245\ \pi cm^3$. Show that the surface area $S\ cm^2$ of the container with the lid containing the hole is given by $S = 49\pi\left(r^2 + \frac{2}{r}\right)$ for $r > 0$.

Find the value of r such the S is minimum.



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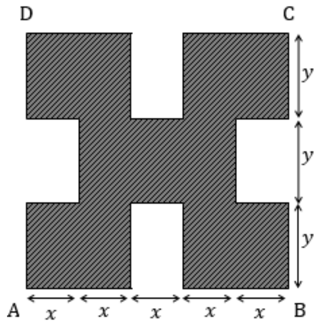

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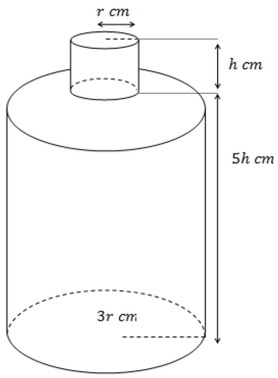
148.

a) Let $f(x) = \frac{16(x-1)}{(x+1)^2(3x-1)}$ for $x \neq -1, \frac{1}{3}$.

[redacted]

[redacted]

b) A bottle with a volume of $391\pi cm^3$ is to be made by removing a disc of radius r cm from the top face of a closed hollow right circular cylinder of radius $3r cm$ and height $5hcm$, and fixing an open hollow right circular cylinder of radius $r\ cm$ and height $h\ cm$, as shown in the figure. It is **given that** the total surface area $S\ cm^2$ of the bottle is $S = \pi r(32h + 17r)$. Find the value of r such that S is minimum.



2018 A/L

149. (a) Let $f(x) = \frac{(2x-3)^2}{4(x^2-1)}$ for $x \neq \pm 1$.

Show that $f'(x)$, the derivative of $f(x)$, is given by

$f'(x) = \frac{(2x-3)(3x-2)}{2(x^2-1)^2}$ for $x \neq \pm 1$.Sketch the graph of $y = f(x)$ indicating the asymptotes, y-intercept and the turning points. Using the graph, find all real values of x satisfying the inequality $\frac{1}{f(x)} \leq 1$.

(b) Let $f(x) = \frac{9\,(x^2 - 4x - 1)}{(x-3)^3}$ for $x \neq 3$.

Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x) = -\frac{9\,(x+3)(x-5)}{(x-3)^4}$ for $x \neq 3$.

Sketch the graph of $y = f(x)$ indicating the asymptotes, y – intercept and the turning points.

It is given that $f''(x) = \frac{18\,(x^2 - 33)}{(x-3)^5}$ for $x \neq 3$.

Find the x – coordinates of the points of inflection of the graph of $y = f(x)$.

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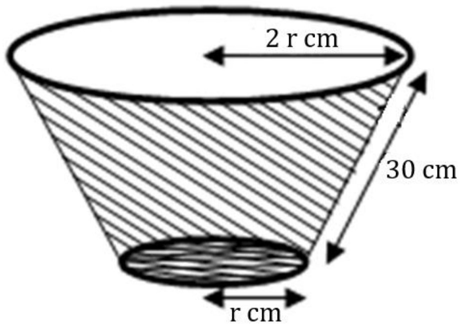

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150.

a. Let $f(x) = \frac{x(2x-3)}{(x-3)^2}$ for $x \neq 3$.

Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x) = \frac{9(1-x)}{(x-3)^3}$ for $x \neq 3$.

Hence, find the interval on which $f(x)$ is increasing and the intervals on which f(x) is decreasing. Also, find the coordinates of the turning point of $f(x)$.

It is given that $f''(x) = \frac{18x}{(x-3)^4}$ for $x \neq 3$.

Find the coordinates of the point of inflection of the graph of $y = f(x)$. Sketch the graph of $y = f(x)$ indicating the asymptotes, the turning point and the point of inflection.

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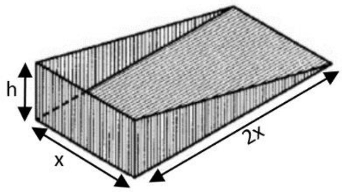

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