MATHSCRIBER

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A/L Combined Maths

CENTRE OF MASS

Raj Wijesinghe

Centre of Mass

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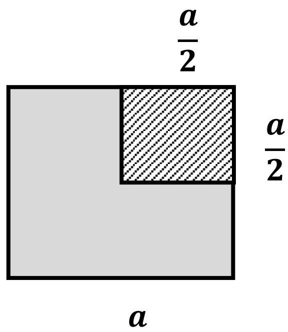

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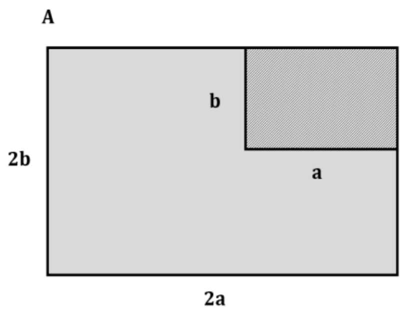
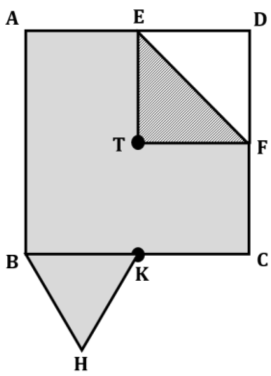


Centre of Mass

[redacted] The triangular lamina BKH is joined to the square lamina ABCD and a triangular lamina EFD is folded down [redacted] with the side length of BK $= \frac{a}{2}$ and the triangle ETF is an isosceles triangle with the side length of ET $= \frac{a}{2}$.



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11. Use integration, to find
    i. The position of the centre of mass of a uniform hemispherical shell of radius a and area density is ρ.

    ii. The mass and the position of the centre of mass of a right uniform hollow cone of height h, and semi vertical angle α, where the area density of the hollow cone is kρ.

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17. A hollow body made of a thin material consists of a right circular cone with vertical angle 2α is joined through the common circular base of the hemisphere and the cone. The cone and the hemisphere are adjoined by side of their circular common bases. Find the centre of mass of the composite object. Furthermore, show that, if $6\cos\alpha = \sqrt{37} - 1$, this hollow body can rest in equilibrium on a smooth horizontal plane, with the curved surface touching at any point of the hemisphere on the plane.

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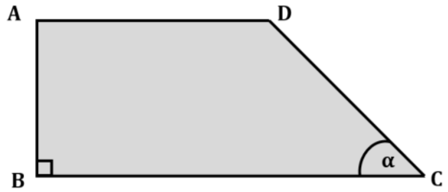

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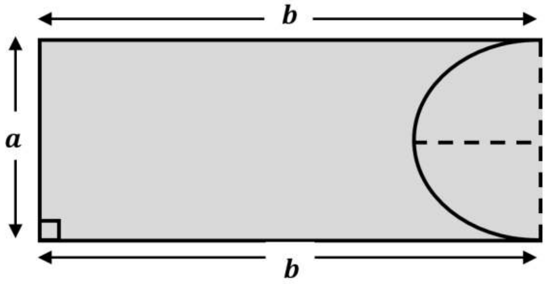

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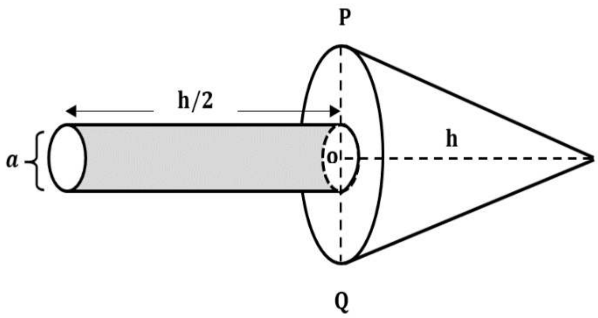
20. The height of the right uniform solid cone is h and the radius OP $= a$.

A solid is made up from a uniform solid right circular cone of radius a and height h and a cylinder of diameter a and length $\frac{h}{2}$ as shown in the diagram. The density of the material of the handle (cylinder) is ρ and the density of the material of the cone is σ . Show that the distance of the centre of mass of the composite solid body is $\frac{3h\ (p + 2\sigma)}{4\ (3p + 2\sigma)}$ from the plane face of the cylinder.



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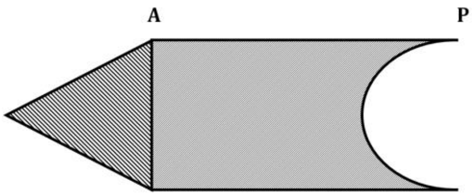

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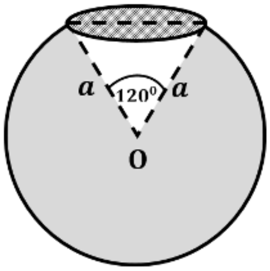
22. Find the position of the centre of mass of a solid hemisphere and a solid right circular cone from the plane of support.



A solid object of density of ρ is made as a remainder, after cutting through a solid sphere by a plane as shown in the diagram. If the radius of the sphere is a, show that the mass of the remainder is $\frac{\pi a^3 \rho}{24}\ (8\pi + 3\sqrt{3})$. Show also that the distance of the centre of mass is $\frac{3\sqrt{3}a}{8\pi + 3\sqrt{3}}$ from O.

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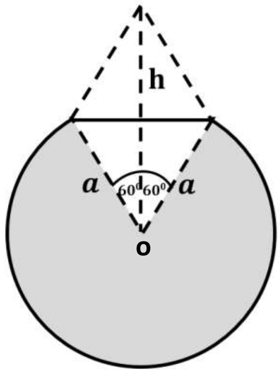
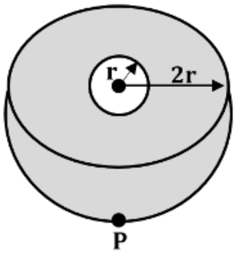
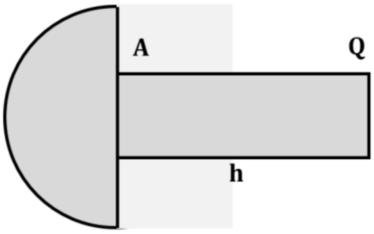


23. Show that the position of centre of mass in a hollow hemi sphere of radius a is at a distance $\frac{a}{2}$ along the symmetric axis of the hemisphere.

As shown in the diagram a circular plane of the hollow hemisphere of radius $2r$ is covered by plate such that [redacted]



As shown in the diagram a uniform hollow circular cylinder with one closed side (side Q is closed) is attached to the earlier object such that both open ends are welled together, the radius and the height of the cylinder are r , h respectively. The cylinder is made from the same material as the composite object. When the system is freely hung from Q, find the angle of which the line AQ makes with the downward vertical.



24. A uniform circular hollow cone has radius r and a slanted height l, area density of the cone is σ. Use integration to show that the mass of the hollow cone is given by $\pi l r \sigma$.

A frustum of height h which is made from a uniform hollow circular cone has radii r_1, r_2 at each of its ends $(r_2 > r_1)$. Show that the distance for the centre of mass of the frustum is at a hypothetical distance $\frac{2h}{3}\left(\frac{r_2}{r_2 - r_1}\right)$ from the vertex of the cone.

Now the frustum is freely hung from a point on the circular edge which has a radius r_2. Find the angle of which has the circular plane (plane with radius r_2) makes with the downward vertical.

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30. Use integration to find center of gravity of a uniform solid cone of radius a and height h. Furthermore, use integration and find the centre of gravity in solid hemisphere of radius a.

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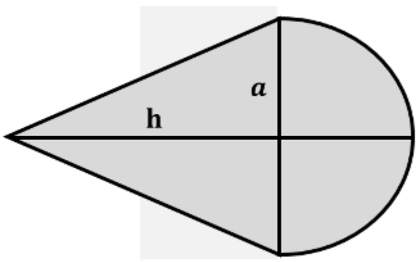

i. [redacted] $\frac{3a^2\,\sigma - \mathrm{h}^2\rho}{4(\mathrm{h}\rho + 2a\sigma)}$ from the common base along the symmetrical axis.

ii. If the composite object is at equilibrium while any point of the curved surface of the solid hemisphere is touching a horizontal floor, for the above condition to be satisfied, show that $\mathrm{h} = \sqrt{\frac{3\sigma}{\rho}}\,a$.

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34. Use integration and show that the centre of mass of a solid hemisphere of radius a is at a distance $\frac{3a}{8}$ away from its surface of the circular plane along the symmetrical axis.

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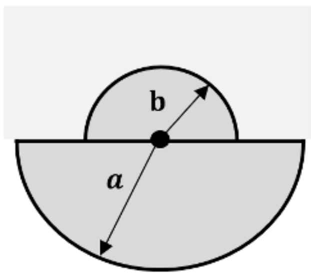

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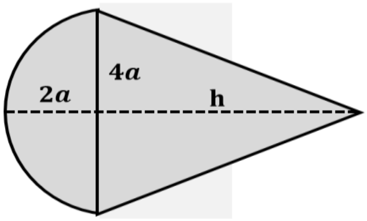
36. If a solid spherical zone is constructed by a material which has a density of ρ, Use integration and show that the mass of a solid spherical zone which has a radius $5a$ and a height $2a$ is $\frac{52\pi a^2\rho}{3}$.

   i. Show that the centre of mass of the spherical zone is at a distance $\frac{9a}{13}$ away from the plane surface along the symmetrical axis of the solid spherical.

   ii. Now a right uniform solid cone of base radius $4a$ and height h is attached to the spherical zone such that both objects have a common base. Show that the centre of mass of the composite object is at a distance of $\frac{h^2-9a^2}{13a+4h}$ away from the common base along the symmetric axis of the composite object.

If the composite object is at equilibrium while the curved surface of the spherical zone is touching a rough horizontal floor, show that h $< 2(6 + \sqrt{21})a$.



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39. A solid spherical zone of height $\frac{a}{2}$ is cut off from a sphere of radius a show that the centre of mass of the spherical zone is at a distance $\frac{27a}{40}$ from the hypothetical centre of the sphere.

Now a right uniform solid cone of base radius $\frac{\sqrt{3}a}{2}$ and a height h is attached to the spherical zone such that both objects have a common base. Find the centre of mass of the composite object, for the composite object to hold its equilibrium while the curved surface of the solid spherical zone touching a

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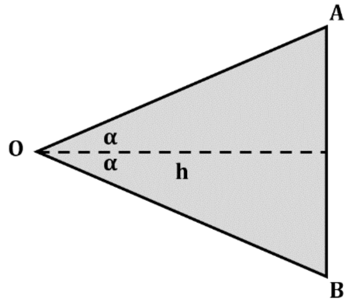

42. Use integration to find the centre of mass of a right uniform hollow cone which has a base radius a and a height h.

Now a bucket is made from a thin metal plate by covering the small circular plane of a frustum of height h which was made by a right uniform hollow cone. The radiuses of circular planes at each end of the frustum are a and $2a$. Show that the distance along the symmetrical axis the centre of mass of the bucket from the small circular end is $\frac{5lh}{(9l+3a)}$. Where l is the slanted height of [redacted]

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46. Find the centre of mass of a semi-circular arc of radius a and centre O.
A rod of length $2l(> 2a)$ is fixed to the axis of a semi - circular arc of radius a and mass M such that the composite object takes the shape of an anchor of a ship. Now this anchor is placed on a rough horizontal surface such that its curved surface is touching the horizontal floor. Initially the anchor remains in rest at a position of equilibrium symmetrically, such that the horizontal surface touching the curved surface of the semi - circular arc. Now a small angular displacement is given to the anchor, if the anchor once again reaches its original position show that the inequality $\mathrm{M} > \frac{\pi \mathrm{m}(l-a)}{2a}$ is satisfied.

47.

i. A uniform rod AB of weight W and length $2a$ is kept in equilibrium with the end A in contact with a rough vertical wall; it is supported by a light inextensible string of equal length $2a$ connecting the other end B to a point C of the wall, vertically above A. The rod is inclined at an angle θ to the upward vertical and it lies in a vertical and it lies in a vertical plane perpendicular to the wall. Find the tension in the string and show that $\theta \geq \cot^{-1}\left(\frac{\mu}{3}\right)$, where μ is the coefficient of friction.

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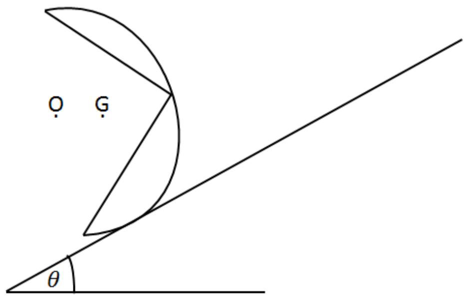
2001 A/L

49. Show that the centre of mass of a uniform solid hemisphere of radius a is at a distance $\frac{3}{8}a$ from the base of the hemisphere.

A solid is formed by the removal of a right circular cone of base radius a and height a from a uniform solid hemisphere of radius a. The plane bases [redacted]

[redacted] horizontal. O and G are in the same vertical plane through a line of greatest slope of the plane. Given that OG is horizontal, show that θ = 30°.



If W is the weight of the hemisphere, obtain in terms of W, the values of the fictional force and the normal reaction at the point of contact.

Find also the smallest possible value of the coefficient of friction between the plane and the solid.

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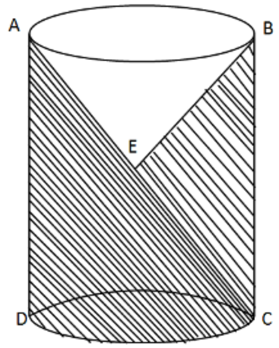

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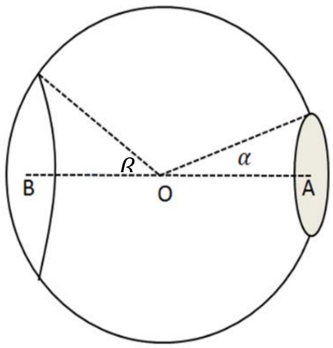

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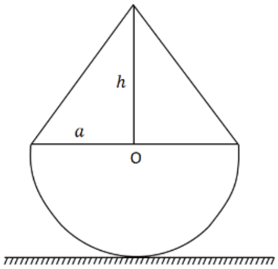
53. The body shown in the figure consists of a uniform solid hemisphere of centre O and radius a, and a uniform solid right circular cone of the same density with base radius a and height h, rigidly joined at the common base.



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displaced from this position of equilibrium so that the axis of symmetry makes a small angle with the vertical. Show that the body will topple over, if $h > \sqrt{3}a$. What happens if,

i. $h < \sqrt{3}a$

ii. $h = \sqrt{3}a$

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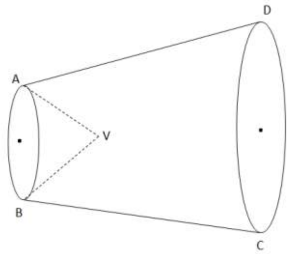
55. In the figure below, ABCD represents a uniform solid body of density ρ in the form of a frustum of height h, of a right circular cone. The diameters of



its circular plane faces are $AB = 2\lambda a$ and $CD = 2a$, where λ is a parameter and $0 < \lambda < 1$.

Show, by integration, that its mass is $\frac{1}{3}\rho\pi a^2\, h\,(1 + \lambda + \lambda^2)$, and that its centre of mass G is at a distance $\frac{h}{4}\frac{(3+2\lambda+\lambda^2)}{1+\lambda+\lambda^2}$ from the centre of the smaller face.

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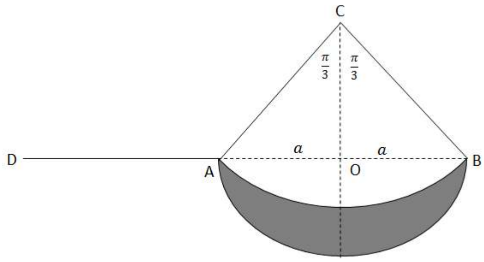

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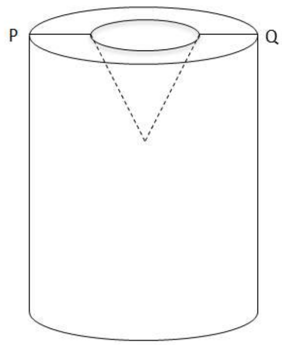

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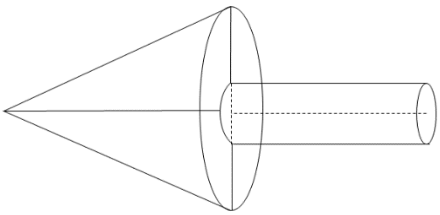

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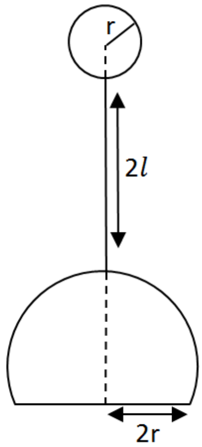

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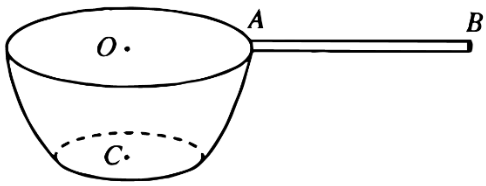

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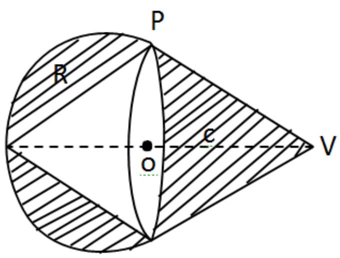
62. Using integration, find the positions of the centre of mass of a uniform solid cone of base radius a and height h, and a uniform solid hemisphere of radius a.

Let R denote the solid body obtained by removing a right circular cone C, of base radius a and height a from a uniform solid hemisphere of mass M, radius a and centre O. Find the mass of the solid body R, in terms of M, and the position of its centre of mass.

The solid cone C is next fixed to the solid body R so as to form a composite body S, as shown in the figure. The circular edge of the base of C is rigidly attached to the rim of R, so that the centre O of the rim is coincident with centre of



the base of C. Show that the centre of gravity G of the composite body S is on its axis of symmetry at a distance $\frac{a}{8}$ from the common centre O of the bases.

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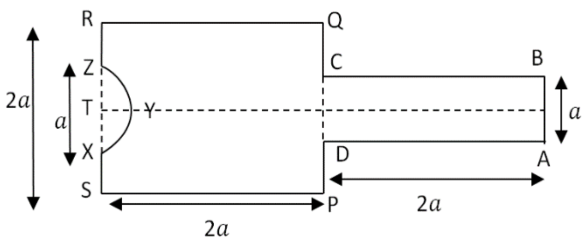

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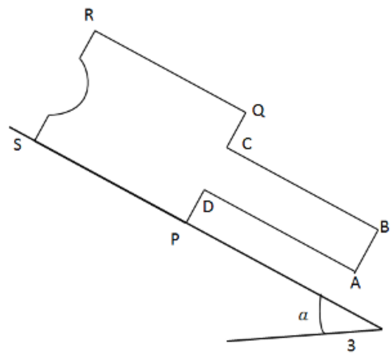

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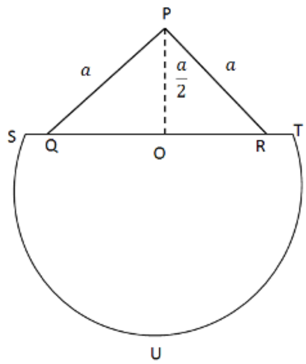

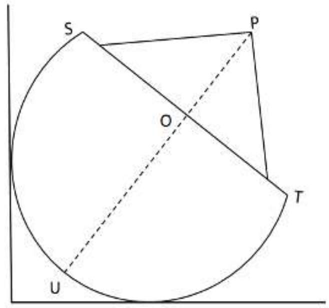

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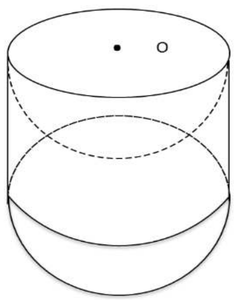

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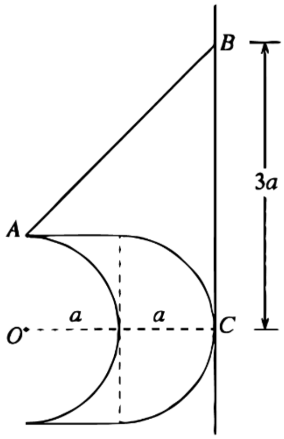

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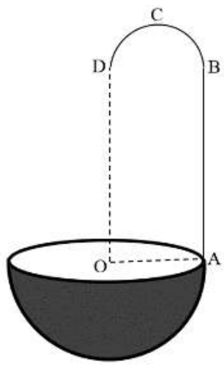

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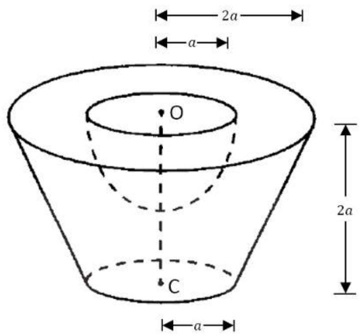
67. Show that the centre of mass of

  i. a uniform solid right circular cone of base radius r and height h is at a distance $\frac{h}{4}$ from the centre of the base,

  ii. a uniform solid hemisphere of radius r is at a distance $\frac{3r}{8}$ from its centre.



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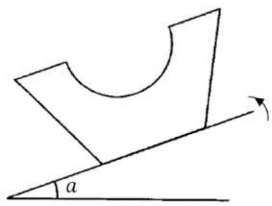

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