MATHSCRIBER

A/L Combined Maths

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Raj Wijesinghe

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[redacted] If the circle $x^2 + y^2 = 9$ and straight line $y = \sqrt{3}x + c$ touches tangentially, [redacted]

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20. Show that the point P(4, -1) lies on the circle $S = x^2 + y^2 - 2x - 6y - 15 = 0$. Find the equation of the tangent drawn to the circle at the point P.

21. Show that the point Q(-2, 7) lies on the circle $S = x^2 + y^2 - 2x - 6y - 15 = 0$. Find the equation of the normal drawn to the circle C at the point Q.

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Find the position of p(2,-3) relative to the circle $x^2 + y^2 - 2x - 4y - 5 = 0$

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Shade that the area satisfied by $x^2 + y^2 - 4x - 6y - 3 \geq 0$ and $x^2 + y^2 - 2x - 2y - 7 \leq 0$ in the XOY plane.

27. Find the equation of the tangent chord to the circle $x^2 + y^2 + 4x - 6y + 9 = 0$ relative to the point P(1, -2).

28. If the tangent chord drawn from the variable point P to the circle $S = x^2 + y^2 + 2x + 4y + 1 = 0$ is always going through the point Q(2,-3), show that the path of the point P is $3x - y - 3 = 0$.

29. The tangent chord from the variable point P to the circle $S_1 = x^2 + y^2 - 16 = 0$ always touches the circle $S_2 = x^2 + y^2 - 4 = 0$. Show that the path of P can be shown by the equation of the circle $x^2 + y^2 - 64 = 0$.

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37. Find the equations of the tangents to the circle $x^2 + y^2 - 9 = 0$ from the point P (-2,-3).

38. Find the equation of the tangent chord to the circle $S = x^2 + y^2 - 6x - 6y + 9 = 0$ from the point P(-3, 3).

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41. The tangent drawn to the circle S_1 at the point $P\left(\frac{-4}{5}, \frac{12}{5}\right)$ touches the circle S_2 at the point $Q\left(\frac{4}{5}, \frac{-12}{5}\right)$. The centers of S_1 and S_2 lies on the line $y = -x$. Find the equations of S_1 and S_2.

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56. Find the equation of the circle which is orthogonal to the circle $S = x^2 + y^2 + 2x + 3y + \frac{15}{2} = 0$ and passing through the intersection points of the circles $S_1 = x^2 + y^2 + 2x + 6y - 4 = 0$ and $x^2 + y^2 - 9 = 0$.

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77. Find the equation of the straight line which passes through the intersecting point of the lines l_1, l_2 given in 76) and having an intercept of $\frac{5}{3}$.

78. Find the equation of the straight line which passes through the intersection point of the lines l_1, l_2 given in 76) and parallel to the line $7x - 3y + 2 = 0$.

79. Find the equation of the straight line which passes through the intersection point of the lines l_1, l_2 given in 76) and perpendicular to the line $7x + 8y + 3 = 0$.

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83. Do the points (2, -3), (-1, 5) lie on same side or opposite sides relative to the straight line $x - y + 1 = 0$.

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103. Find the equation of the circle S_2 of center Q(8, 0) and touches the circle $S_1 = x^2 + y^2 - 12y + 11 = 0$ externally.

104. The S_1 , S_2 and S_3 are represented by $S_1 = x^2 + y^2 - 4x - 6y - 12 = 0$, $S_2 = x^2 + y^2 - 10x - 6y + 30 = 0$ and $S_3 = x^2 + y^2 - 20x - 6y + 100 = 0$. Show that the circles

   a) S_1 and S_2 touch each other internally.
   b) S_1 and S_3 touch each other externally.
   c) S_2 and S_3 touch each other externally.

   Without solving the equations S_1 , S_2 and S_3, show that the three circles contact each other in a common point.

105. The circles S_1 , S_2 and S_3 are represented by $S_1 = x^2 + y^2 - 9 = 0$, $S_2 = x^2 + y^2 - 12x + 27 = 0$ and $S_3 = x^2 + y^2 - 6x - 6\sqrt{3}y + 27 = 0$ respectively. The points O, A and B respectively are the centers of them.

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109. Find the equation of the circle which intersect the circles $S_1 = x^2 + y^2 + 6x + 4y + 8 = 0$, $S_2 = x^2 + y^2 + 8x + 6y + 12 = 0$ and $S_3 = x^2 + y^2 + 10x - 8y = 0$ orthogonally.

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141. Find the position of the points (2, 2), (1, 3), (1, 1) with respect to the circle $x^2 + y^2 - 8 = 0$.

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149. Find the condition for the two circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x - 2f_2y + c_2 = 0$ to touch each other. If they touch each other, show that the lines $2(g_1 - g_2) + (f_1 - f_2) + c_1 - c_2 = 0$ and $(f_1 - f_2)\,u - (g_1 - g_2)\,y + f_1g_2 - f_2g_1 = 0$

are on each of the two circles. Show that the two circles touch each other externally and find the coordinate of the contact point.

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171. The tangents passing through the points Q_1, Q_2 which are on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ are intersecting at the point $P_0 \equiv (x_0, y_0)$. Show that the equation of the tangent Q_1, Q_2 relative to P_0 is $xx_0 + yy_0 + g(x + x_0) + f(y + y_0) + c = 0$. Prove that the tangents of the circles $x^2 + y^2 + 6y + 5 = 0$ and $x^2 + y^2 + 2x + 8y + 5 = 0$, relative to the point (1,-2) coincide. Further, find the coordinates of the point, such that the tangents of the above circles are same.

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210. The circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ intersects the line $u = ax + by + c = 0$. Explain $S + \lambda u = 0$. Find the requirement for the chord $u = 0$ of the circle $S \equiv 0$ to subtend 90 degrees at the origin.
Hence, show that the path of the base of the perpendicular drawn from origin to PQ is $2x^2 + 2y^2 + 2gx + 2fy + c = 0$ if the variable chord PQ of the circle $S \equiv 0$ subtends a right angle at the origin.

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231. The equation of a circular system is $x^2 + y^2 - 1 + 2\lambda (x - y + 1) = 0$. Here, λ is a parameter. Show that,
   a) The centers of circles are colinear.
   b) There is a common center axis for the system of circles.
   c) The center line is perpendicular to the center axis. Further, show that the equation of the system of circles, orthogonally intersect the system of circles given is $x^2 + y^2 - 2y + 1 + 2\mu (x + y) = 0$. Here, μ is a parameter.

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246. Show that the circles $S_1 \equiv x^2 + y^2 - 2x = 0$, $S_2 \equiv x^2 + y^2 - 10x - 6y + 18 = 0$ touch each other at the point P and find the equation of the common tangent. Show that the line $y + 1 = 0$ is also a common tangent. Find the radius of the circle which intersects the two circles perpendicularly and the center passing through the line $y + 1 = 0$.

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255.

a) Find the possible simplest form of the relationship between 2D cartesian coordinates and polar coordinates.
Find the polar equation of the circle whose center $(\rho \cos\theta, \rho \sin\alpha)$ in cartesian coordinates and radius a. Here, ρ and α are constants.

A circle is moving through three polar coordinates (0, 0), (c, 0) and $(2c, \frac{\pi}{3})$. Find the equation of the circle using polar coordinates.

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256.

a) Find the polar equation of a circle whose radius is a taking a point as a polar point and the diameter along that point as the axis.
Show that the point is positioned on this circle. Show that the equation of the tangent drawn at this point to the circle is $r\cos(\theta - 2\alpha) = 2a\cos^2\alpha$.

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258. S, S′ are two circles which have the radii r, r′ and centers c, c′ respectively. Show that S, S′ internally touch each other if and only if $|r - r'| = CC'$. Write down (without proving) the mandatory and adequate requirement for circles S, S′ to externally touch each other. $S_1 = x^2 + y^2 - 6x + 8 = 0$ and $S_2 = x^2 + y^2 - 4 = 0$ are two circles. Show that $S_1 = 0$, $S_2 = 0$ touch each other externally. Prove that the circle $S_3 = x^2 + y^2 + 2\lambda x - 4(1+\lambda) = 0$ touch the circle $S_2 = 0$ internally and the circle $S_1 = 0$ externally for all real values of λ where $\lambda > -2, \lambda \neq 0$. Find the radius of the circle which is touching the straight line $4x + 3y - 44 = 0$ and touching the circle $S_1 = 0$ internally and the circle $S_2 = 0$ externally. What is the equation of the circle that touch the circles $S_1 = 0$ and $S_2 = 0$ internally.

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269. Find the equation of the line l which has gradient -1 as the and passing through the point A(-2, 6). Show that the one tangent to the circle $x^2 + y^2 = 8$ from the point A is l. Show that the coordinates of the contact point B of the line l and the circle is (2, 2). O is the origin. The base of the perpendicular

drawn from B to AO is C. The contact point of the other tangent to the circle from the point A is D. Find the coordinates of the points C and D. Find the angle BAD.

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275. Show that the circles $S \equiv x^2 + y^2 - 2x - 6y + 1 = 0$, $S' \equiv 3x^2 + 3y^2 - 21x + 2y + 35 = 0$ are positioned completely externally to each other. Find the [redacted]

276. The tangents which are passing through the points Q_1, Q_2 on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, meet at the point $P_0 \equiv (x_0, y_0)$. Show that the equation of the tangent Q_1, Q_2 at the point P_0 is $xx_0 + yyo + g(x + x_0) + f(y + y_0) + c = 0$. Prove that the tangents drawn from the point $(1, -2)$ to the circles $x^2 + y^2 + 6y + 5 = 0$ and $x^2 + y^2 + 2x + 8y + 5 = 0$ coincide. Further, find that there is another point such that the tangents corresponding to the circles mentioned above are same, and find its coordinates.

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278.

a) Show that the polar equation $r = 2a \cos\theta$ represents a circle which is passing through the polar 0 and radius is a. what is the center of that circle? Show that from $r = 2a \cos(\theta - \alpha)$ and $r = 2b \sin(\theta - \alpha)$ represent two perpendicularly intersecting circle. Here, a, b, α are constants.

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286. If the two circles $x^2 + y^2 + 2gx + 2fy + c = 0$ and $x^2 + y^2 + 2g'x + 2f'y + c' = 0$ intersect orthogonally, show that $2gg' + 2ff' = c + c'$. Let P and Q be the points on the circle $S \equiv x^2 + y^2 - a^2 = 0$ with the coordinates

(-a, 0) and (a cos 0, a sin 0) respectively. The chord PQ is extended to a point R so that PQ = QR. Find the coordinates of R and show that, as θ varies, R lies on a circle S'. Obtain the equation of S'. A third circle S'', which touches the y axis, intersects both circles S and S' orthogonally. Show that there are two such circle S'', and obtain their equations.

2003 A/L

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2004 A/L

288. Show that the two circles with equations $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 - 4r^2 = 0$ never touch each other externally but touch each other internally, if $g^2 + f^2 = r^2$. Find the coordinates of the point of contact in the latter case. Show that there are two circles, which pass through the origin and the point (a, 0), where $0 < a < 1$, and touch the circle whose equation is $x^2 + y^2 - 4 = 0$. Find the coordinates of the points of contact. Also find the equation of the circle having these points as ends of a diameter.

2005 A/L

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304. Find the centre and the radius of the circle which passes through the origin O and the two points where the line $y = 1$ intersects the circle $x^2 + y^2 - 2x - 2y + 1 = 0$.

2015 A/L

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309. Let m ∊ ℝ Show that the point P≡ (0, 1) does not lie on the straight-line l given by y = mx. Show that the coordinates of any point on the straight line through P perpendicular to l can be written in the form (-mt, t + 1), where t is a parameter. Hence, show that the coordinates of the point Q, the foot of the perpendicular drawn from P to l. are given by $\left(\frac{m}{1+m^2}, \frac{m^2}{1+m^2}\right)$ Show that, as m varies, the point Q lies on the circle S is given by $x^2 + y^2 - y = 0$, and sketch the locus of Q in the xy-plane. Also, show that the point R ≡ $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}\right)$ lies on S. Find the equation of the circle S' whose centre lies on the x-axis, and touches S externally at the point R. Write down the equation of the circle having the same centre as that of S' and touching S internally.

2017 A/L

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