

A/L COMBINED MATHS



★ MOTION GRAPHS & COLLISION ★

DYNAMICS

RAJ WIJESINGHE



MATHSCRIBER

Motion Graphs

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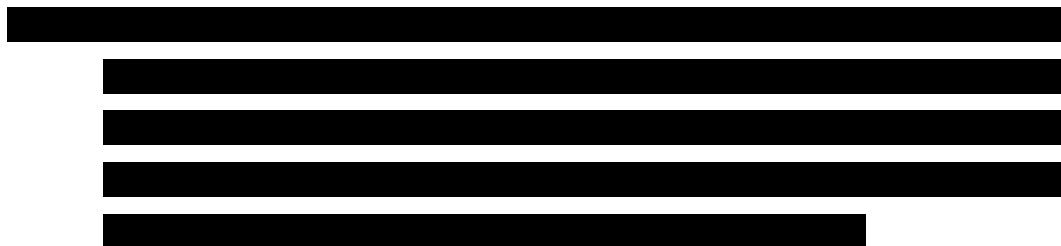
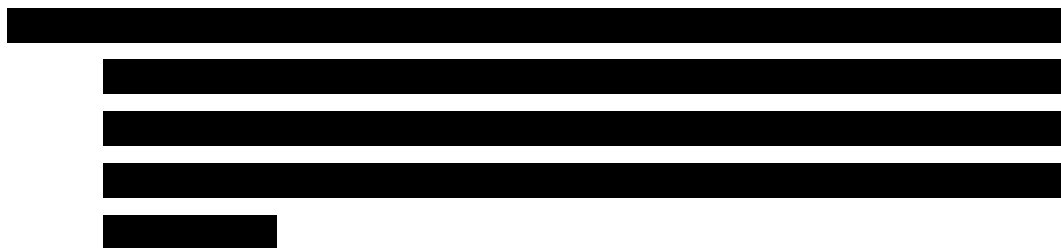
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Motion Graphs



9. A bus is traveling along a straight road with uniform velocity $u \text{ ms}^{-1}$. When the bus is at point A, a passenger indicates that he wants to get down at the halt H which is $4l/m$ ahead. Driver applies breaks step by step at points A, B, C. In the intervals AB, BC, CH the uniform retardations are $a, 2a, 3a \text{ ms}^{-2}$ where $AB = CH = l$ and $BC = 2l$. The bus stops at H. Draw a velocity graph for the motion of the bus. Hence show that $U = 4\sqrt{al}$ Also, show that time taken to travel from A to H is $[24 - 3\sqrt{14} - \sqrt{6}] \frac{2l}{3u}$ seconds.



Motion Graphs

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Motion Graphs

15. The distance between two bus halts is S m. A bus starts from rest at the halt A goes and stops at the halt B. The bus has a maximum acceleration a_1 ms^{-2} and maximum retardation a_2 ms^{-2} . Show that the minimum time taken by bus to travel s m is

$$\left[\frac{2s(a_1 + a_2)}{a_1 a_2} \right]^{\frac{1}{2}}.$$

In the above case maximum velocity of the bus is given as v ms^{-1} . Because of a road repair a notice had been put up to inform that speed should not exceed v_0 ($v_0 < v$). Show that minimum excess time taken by bus to travel from A to B is $S(v - v_0)^2 / v_0 v^2$.

Motion Graphs



22. A balloon starts from rest and rises vertically. In the 6th second of its motion it rises $55\sqrt{3}h$ m. Determine the uniform acceleration and the velocity at the end of the 6th second and distance moved in that time.
23. A motorcycle is travelling with a uniform acceleration. It passes 25th km post with velocity of 15 ms^{-1} . In another 40 seconds time it passes the 26th

Motion Graphs

km post. What is the velocity at the 26th km post. It travels in the same way and passes a bridge with velocity of 75 ms⁻¹. Determine the time taken and distance travelled from 26th km post to pass the bridge.

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Motion Graphs

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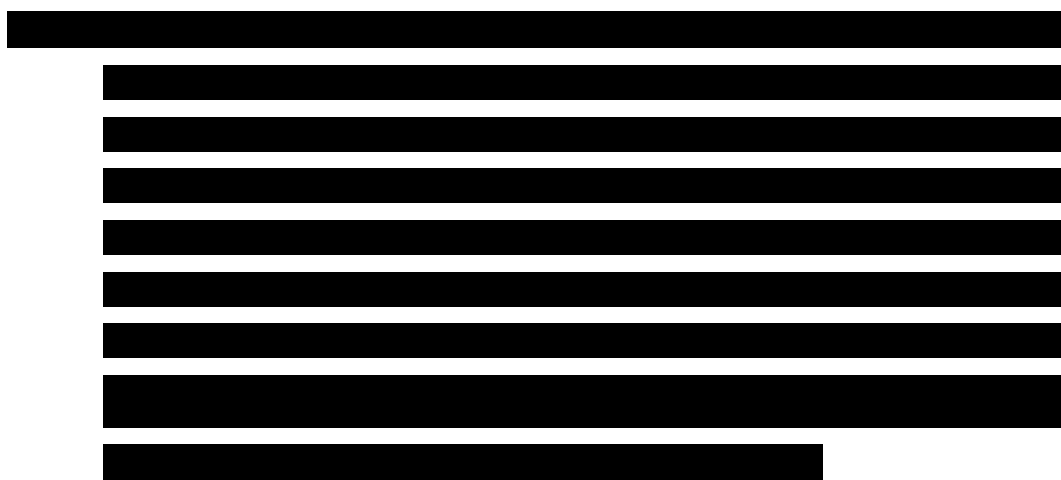
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Motion Graphs



Motion Graphs

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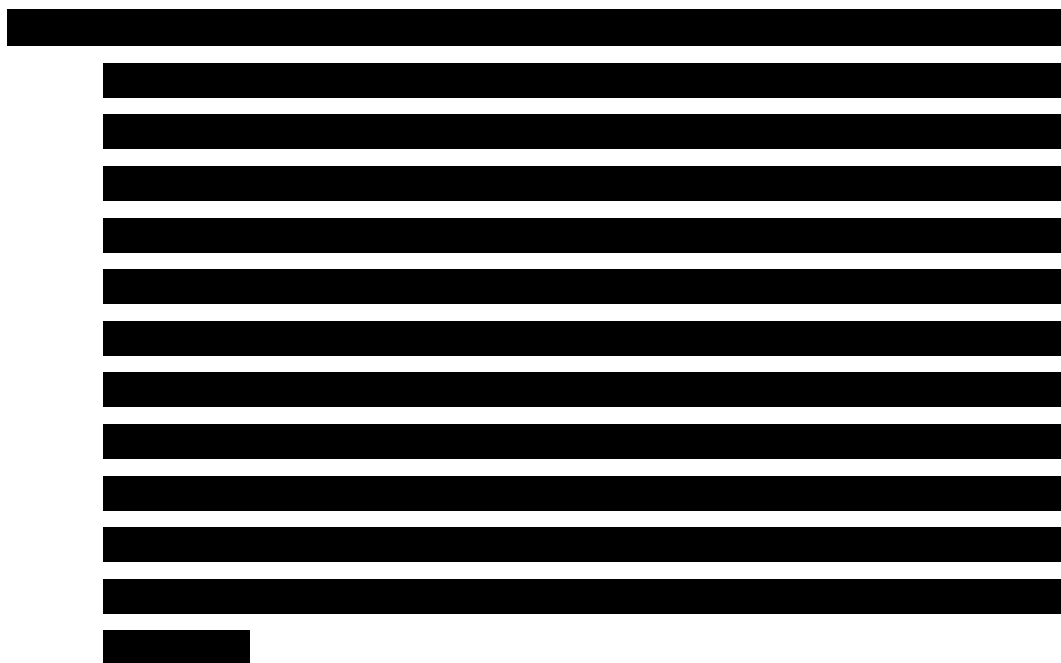
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Motion Graphs



Motion Graphs

43. Two particles A, B start from O and move in a straight line. A starts from rest at O moves with a uniform acceleration of 4ms^{-2} seconds after A started, B starts from O with a velocity of 16ms^{-1} . and moves. Show that they meet twice and find the distance between the two points.

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Motion Graphs

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51. A balloon rises vertically upwards with a uniform acceleration of $g/3$ from rest. When the balloon is at a height a , an object A is released from it. Show that the time taken by A to reach the ground is $\sqrt{\frac{6a}{g}}$. t seconds after releasing A, another object B is released from the balloon. After time t from the moment B is released show that the distance between A and B is $2gt^2$.

Motion Graphs

When A is at its maximum height show that the distance between A and B is $a/3$.



54. At $t = 0$, a particle P is projected vertically upwards with velocity $v \text{ ms}^{-1}$ and moves under gravity. If the maximum height attained by the particle is $H \text{ m}$ show that $H = \frac{v^2}{2g}$.

When P reaches A at a height $H_1 \text{ m}$, in its upward motion, a second particle Q is projected from O vertically upwards with velocity $v \text{ ms}^{-1}$. P and Q collide at A. Show that $\frac{H_1}{H} = \frac{8}{9}$. Acceleration under gravity is $g \text{ ms}^{-2}$. Find the time taken for P and Q to meet.

55. A rocket is launched at $t = 0$, from rest with an acceleration of $\frac{g}{5}$. At $t = T$, because of a defect in the engine, pilot detaches himself and uses a parachute. Opening up of the parachute, failure of the engine and changing of the direction of pilot's velocity happen at the same instant. Rocket and pilot come down to the horizontal plane from which they took off at the same moment. When the parachute touches the starting plane show that its velocity is $\frac{gT}{(1 + \sqrt{6})}$.



Motion Graphs

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ii. A particle similarly projected falls to the bottom of a 49 m deep well

Find the time taken

61. A balloon starts rising vertically upwards from rest at point O with constant acceleration 5 ms^{-2} . 30 s after the balloon launch, a ball is released from the balloon. Find the time taken by the ball to reach the ground, and its velocity at that instant.

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Motion Graphs

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64. A body is projected vertically upwards at $t = 0$ with velocity u . The times taken by it to pass the point A, h m above are t_1 in its upward motion and t_2 in its downward motion. Show that $t_1 t_2 = \frac{h}{g}$ and $t_2 - t_1 = \frac{1}{g} \sqrt{u^2 - 2gh}$.
- $g = 10 \text{ ms}^{-2}$

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Motion Graphs

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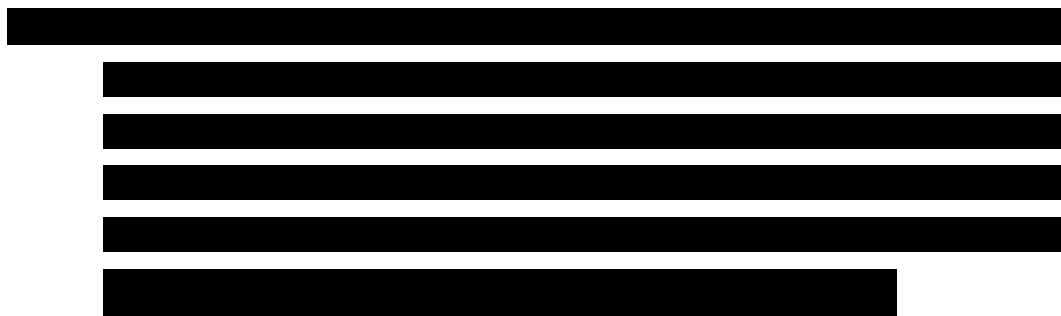
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Motion Graphs

71. Body A released from the top of a tower of height h falls under gravity. At the same instant body B is projected vertically upwards from the bottom of the tower with the velocity just enough to reach the top of the tower. Find the time taken by them to meet and the point at which they meet.



74. A balloon is released at $t = 0$ from a point O on the ground. It rises vertically upwards with constant acceleration $f \text{ ms}^{-2}$. A body A which started from O at $t = T$ moves vertically upwards with an initial velocity $u \text{ ms}^{-1}$ and a constant retardation of $\frac{f}{2} \text{ ms}^{-2}$. On the same diagram draw the velocity time graph for the balloon and the body A . If A just touches the balloon prove that $u = \frac{fT\sqrt{3}}{2}$ show that when the body is at its greatest height, the height of the balloon from the grounds is $\frac{f}{2} \left(\frac{2u}{f} + T \right)^2$.

Collision

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Collision

82. A smooth sphere of mass m moves 'a' distance along a smooth plane inclined 30° to the horizontal and it hits on the smooth horizontal plane and rebounds. The ball hits for the second time on a point at a distance of $\frac{a}{3}$ on the smooth horizontal plane. Obtain the impulse of the first collision.
83. Two particles of mass $2m$ and m move in the same direction with velocities $2u \text{ ms}^{-1}$ and $u \text{ ms}^{-1}$ respectively collide with each other on the smooth horizontal plane and merged and move towards the same direction as the original motion with $V \text{ ms}^{-1}$ velocity. Show that $V = \frac{5U}{3}$. Find the energy loss due to the collision.

[Redacted solution for question 83]

85. A wooden log of mass M kg rests on a rough horizontal plane. A bullet of mass m kg moves along the plane at a velocity of $3u \text{ ms}^{-1}$, and hits on M and gets carved into it. Find the velocity the composite object obtains. (Take dynamic coefficient friction as $\frac{1}{4}$).

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Collision

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89. Water exits horizontally at a velocity of $u = 15 \text{ ms}^{-1}$ through a pipe with a cross-sectional area of 5 cm^2 and hits to a vertical wall. If the mass of 1 m^3 of water is 1000 kg , find the impulse force on the wall.

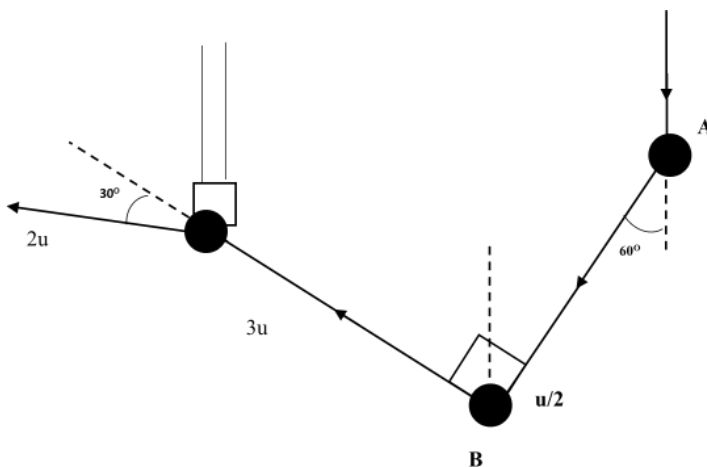
90. A smooth small ball is released from rest from a height of h under gravity, and hits on O at the edge of an α inclined plane to horizontal and rebounds horizontally. The ball hits on a point P , $\frac{h}{2}$ horizontally away and h below. Show that $\alpha = \tan^{-1}\left(\frac{1}{4}\right)$, then find the impulse on the ball by the plane.

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Collision



94. A bullet of mass $m \text{ kg}$ moving at a velocity of $u \text{ ms}^{-1}$, gets carved into a block of wood of mass of $M \text{ kg}$. The block is able to move in the same direction as the bullet. Show that the loss of kinetic energy is $\frac{mMu^2}{2(m+M)}$. Then, another bullet with same mass moving at the same speed as the first bullet hits on the block. Show that the further energy loss is $\frac{mM^2u^2}{2(M+2m)(M+m)}$. Obtain the impulses on the bullets at each condition.

95. A particle of mass $m \text{ kg}$ moving face to face horizontally at a velocity of $u \text{ ms}^{-1}$ hits and gets embedded into an object of mass $M \text{ kg}$ moving horizontally at a velocity of $v \text{ ms}^{-1}$. Find the velocity of the composite object and the impulse between the objects. Show that the energy loss is $\frac{mM(v-u)^2}{2(m+M)}$.

i. $v = 2u$

Collision

- ii. $v = u$, deduce the energy loss in the given instances.
- iii. What can be said about the second condition?

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Collision

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Collision

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106. Two particles A and B of mass $2m$ and $3m$, are attached to an inextensible string and wrapped around a fixed smooth pulley and the system is released from rest. At t time, when the moment u moves at velocity, particle A picks up a immovable C particle, whose mass is $2m$. Show that the velocity at which the particles move as the string shakes is $\frac{gt}{7}$, and the impulsive tension of the string is $\frac{6mgt}{35}$. Show that the time taken for the A and C composite particle to come to rest instantly is t .

Collision

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123. Upul of mass $2M$ kg who is standing on a smooth horizontal plane throws a ball of mass m kg at a velocity v ms^{-1} at an angle of 30° to the horizontal. Kavith of mass $2m$ kg who is also standing still on the same plane, catches the ball at the projected height. Show that the velocity of motion of Kavith is $\frac{Mv\sqrt{3}}{3(m+2M)} ms^{-1}$.

124. An object of mass of $3m$ kg moves in a straight line at a velocity v ms^{-1} . The object explodes into two pieces of mass m kg and $2m$ kg. Both of those pieces move in the same straight line. Explosion increases the energy of the system by $3mv^2$. Find the velocities of the pieces after the explosion.

State the principles of conservation of momentum. A bullet of mass M moving at a velocity u explodes into two pieces. One of these

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Collision

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Collision

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133. A particle with a mass of $3m \text{ kg}$ rests on a smooth plane. The inextensible string attached to the particle is wrapped around a smooth pulley above the table, and particle of mass $2m \text{ kg}$ is attached to the other end, and the system is in equilibrium. The $2m \text{ kg}$ particle is lifted vertically to a height of $h \text{ m}$ and releases under gravity. Show that the velocity at which $3m \text{ kg}$ starts to rise is $\frac{2}{5}\sqrt{2gh} \text{ ms}^{-1}$. Show that the impulsive tension of the string is $6m\frac{\sqrt{2gh}}{5} \text{ Ns}$ and the energy loss of the system is $\frac{6mgh}{5} \text{ J}$.

Collision

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137. Two smooth spheres of masses m and $3m$ are moving in same direction and collide directly. Between the spheres, the initial relative velocity is u and the coefficient of restitution is e . Show that the impulse between the spheres is $\frac{3m(1+e)u}{4}$ and show also that the loss of kinetic energy is $\frac{3mu^2(1-e^2)}{8}$.

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Collision

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Collision

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Collision

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149. A smooth sphere strikes vertically with speed u , on to a smooth horizontal plane which is stationary. Find the speed of rebound after the third impact and the time of flight until the third impact.

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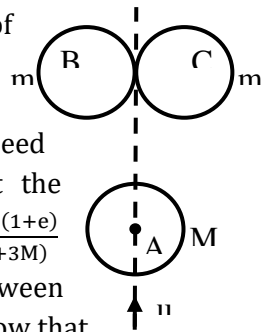
Collision

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153. Three smooth spheres A, B and C of equal radius and of mass M , m and m respectively, are placed on a smooth table. Initially B and C are contacting each other at rest as shown in the diagram and the sphere A is projected with speed u along the common tangent of B and C. Show that the magnitude of the velocity of each sphere B or C is $\frac{\sqrt{3}uM(1+e)}{(2M+3m)}$ after the impact where e is the coefficient of restitution between any two spheres. Find the velocity of A after the impact. Show that $\frac{3mM(1-e^2)u^2}{2(2M+3m)}$ amount of original kinetic energy is lost due to the impact.



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Collision

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157. A smooth sphere of mass m kg strikes vertically with speed u on to a smooth plane inclined at an angle 30° to the horizontal. Given that $e = \frac{1}{2}$ and after the impact the sphere is moving at an angle α° to the horizontal with v velocity find α and show that $v = \frac{u\sqrt{7}}{4}$. Show also that the impulse on the sphere is $\frac{3\sqrt{3}mu}{4}$.

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Collision

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162. A smooth circular tray of radius a has a small vertical edge around the circumference of the tray. A small smooth sphere projected with speed v with an angle α to the horizontal at a point A on the circumference, strikes at B then at C of the edge and reaches A again. Show that $\tan^2 \alpha = \frac{e^3}{1+e+e^2}$ where e is the coefficient of restitution. When the sphere is projected from the point A at an angle 30° to the radius of the circle find a condition for this collision process to be possible.

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Collision

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166. A smooth sphere falls under gravity from a height h , strikes a smooth inclined plane at an angle 30° to the horizontal. The coefficient of restitution is $\frac{1}{4}$. Given that B is the point where the third impact takes place on the inclined plane, show that $OB = \frac{105}{128} h \sin \alpha$.

167. Two smooth uniform spheres each of mass M kg and radius a m are suspended on a horizontal line at two fixed points A and B respectively, by means of two inextensible strings of equal lengths where $AB = 2a$ m. At the instant when the spheres are gently touching each other in equilibrium, a smooth sphere of mass m kg and radius b m moving vertically with velocity u strikes A and B symmetrically. Find the velocities of the spheres after collision, the impulse and show that the loss of kinetic energy is $\frac{mM(1-e)u^2 \cos^2 \alpha}{m \sin^2 \alpha + 2M \cos^2 \alpha}$.

Given that $a = b$, deduce that the loss of kinetic energy.

Collision

168. A sphere of mass m hangs in equilibrium from a fixed point by means of a light inextensible string. A second sphere of mass m' falling vertically strikes the first sphere in a position where the line of centres makes an acute angle α with the vertical. The velocity of the second sphere immediately before impact is v . Given that the coefficient of restitution is e , show that the first sphere starts to move with a velocity of magnitude $\frac{m'(1+e)v \sin \alpha \cos \alpha}{m+m' \sin^2 \alpha}$. Find the impulse between the spheres and the impulsive tension in the string.

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Collision

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Collision

182. Two smooth spheres of equal radius but of different masses moving with velocity 15 ms^{-1} in opposite directions, collide directly. They move with velocities 3 ms^{-1} and 15 ms^{-1} in opposite directions after the collision. Show that the ratio of the masses of the spheres is $5 : 3$. The two spheres collide directly moving in the same direction. The velocity of heavier sphere is 15 ms^{-1} and the lighter sphere is 10 ms^{-1} . Find the velocity of each sphere after the impact, impulse and the loss of kinetic energy.

Collision

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Collision



192. A smooth sphere of mass $2m$ collides directly with a smooth sphere of mass $3m$ which is at rest. Due to the collision, $\frac{1}{3}$ of the original kinetic energy is lost. Find e , the coefficient of restitution. Obtain I , the impulsive reaction between the spheres.

193. A smooth sphere of mass m moving with velocity $u \text{ ms}^{-1}$ strikes directly, a smooth sphere of mass km which is moving with velocity $\lambda u \text{ ms}^{-1}$ in the same direction. Given that the first sphere comes to rest after the collision, show that the coefficient of restitution between the spheres is $\frac{1+k\lambda}{k(1-\lambda)}$. Deduce that $k > 1$ for this process to be possible. Find the impulse between the spheres and obtain the loss of kinetic energy.



195. A sphere of mass $3m$ moving with velocity u directly strikes a sphere of mass $2m$ which is at rest. After the collision, the second sphere moving with velocity v collides a third sphere which is moving towards second sphere with velocity kv . The third sphere comes to rest due to the collision. The coefficient of restitution for all collisions is e . Show that $v = \left(\frac{1+e}{3}\right)u$ and $k = \frac{2(1+e)^2}{3(1-2e)}$.

Collision

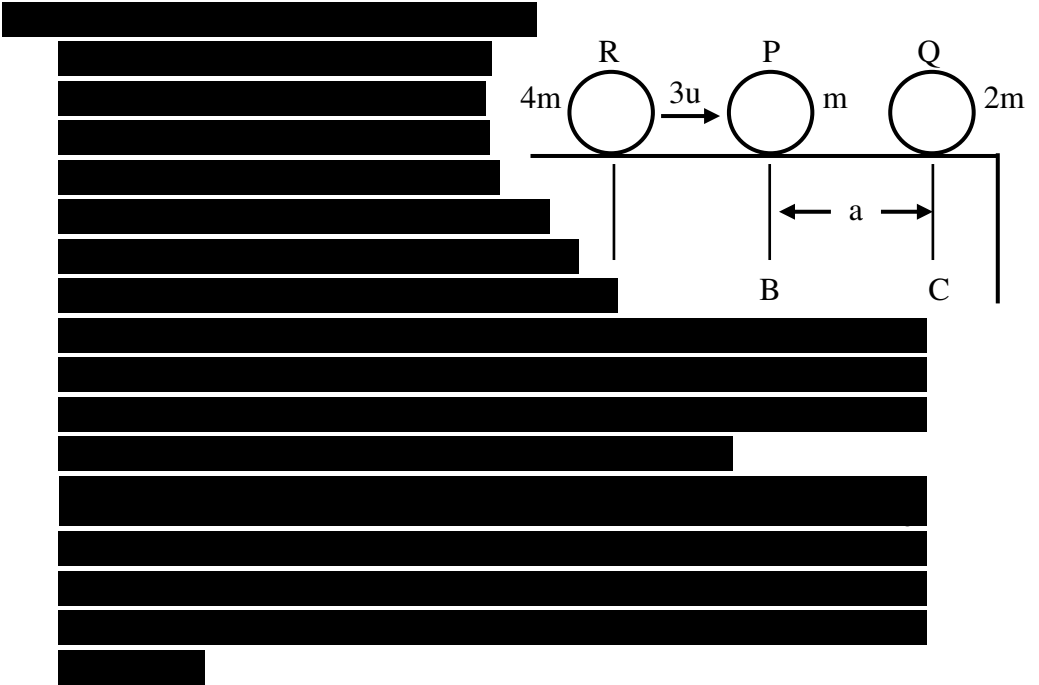
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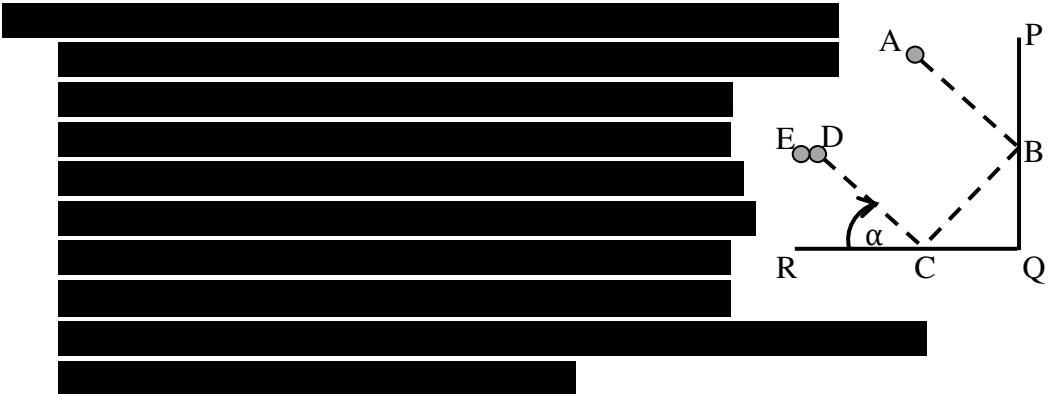
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Collision



201. A smooth sphere A, of mass $2m$ moving with speed u on a smooth horizontal plane collides directly with a stationary smooth sphere B, of mass m of equal radius. The coefficient of restitution is e . After collision sphere B then strikes a smooth perfectly elastic vertical wall normally and rebounds from the wall to collide directly with sphere A again. Show that the velocity of B is then $\frac{2u(1+e)^2}{9}$ and find the velocity of A. Coefficient of restitution is e .



Motion Graphs Past Papers

203. A train, 100 metres long, starts from rest at a station A and moves with constant acceleration. Later, it takes 10 seconds to pass a signal post B. The train is moving with velocity 11 ms^{-1} when the rear of the train passes B. Draw a velocity - time graph for the motion of the train.

Using this graph or otherwise,

- find the velocity **with which** and the time at which the front of the train passed B.
- find the acceleration of the train and show that the total distance travelled is 302.5 metres, when the rear of the train is at B.



Motion Graphs Past Papers

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Motion Graphs Past Papers



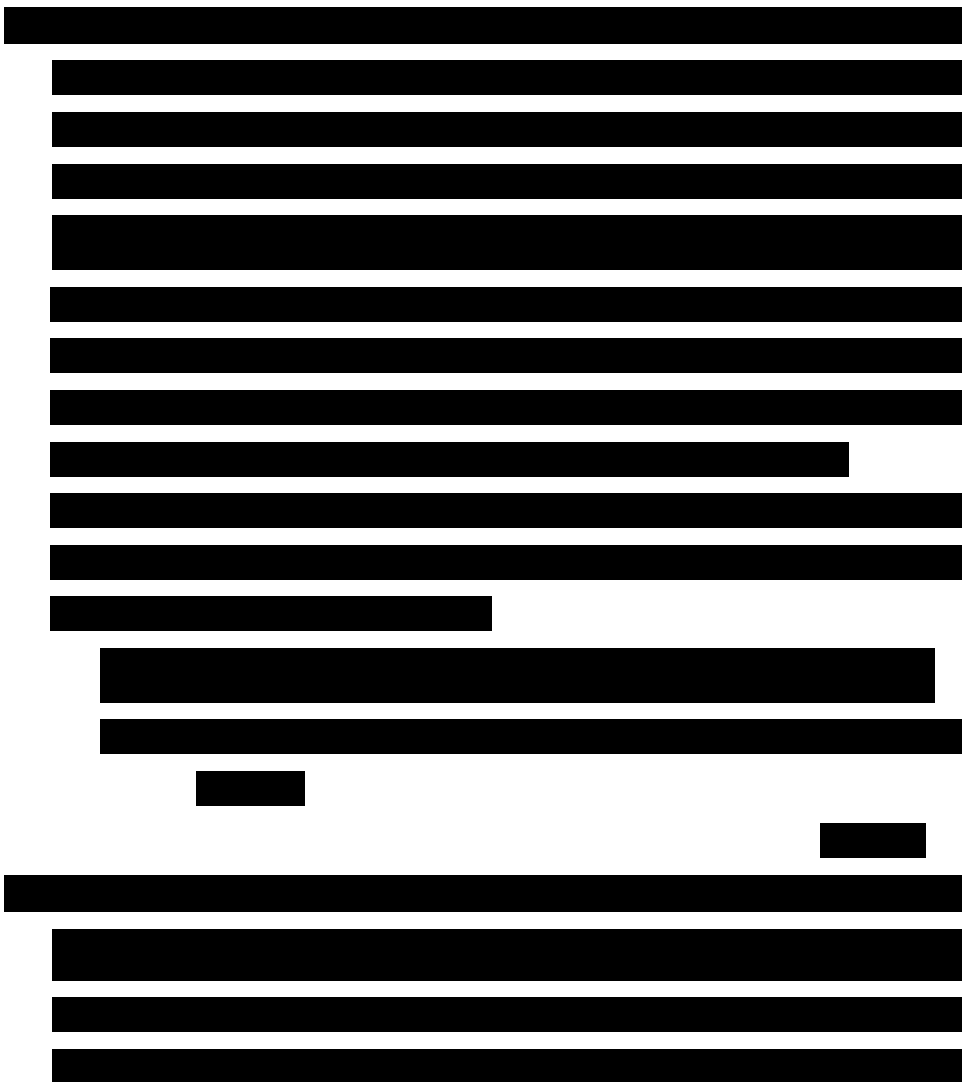
212. A balloon is rising with constant velocity U , relative to earth. At time $t = 0$, particle P is projected from the balloon, vertically upwards, with velocity V relative to the balloon. At time $t = t_1$ another particle Q is projected from the balloon, vertically upwards, also with velocity V relative to the balloon. The two particles P and Q meet at time $t = t_2$. Sketch the velocity-time graphs, separately, for the motion of

Motion Graphs Past Papers

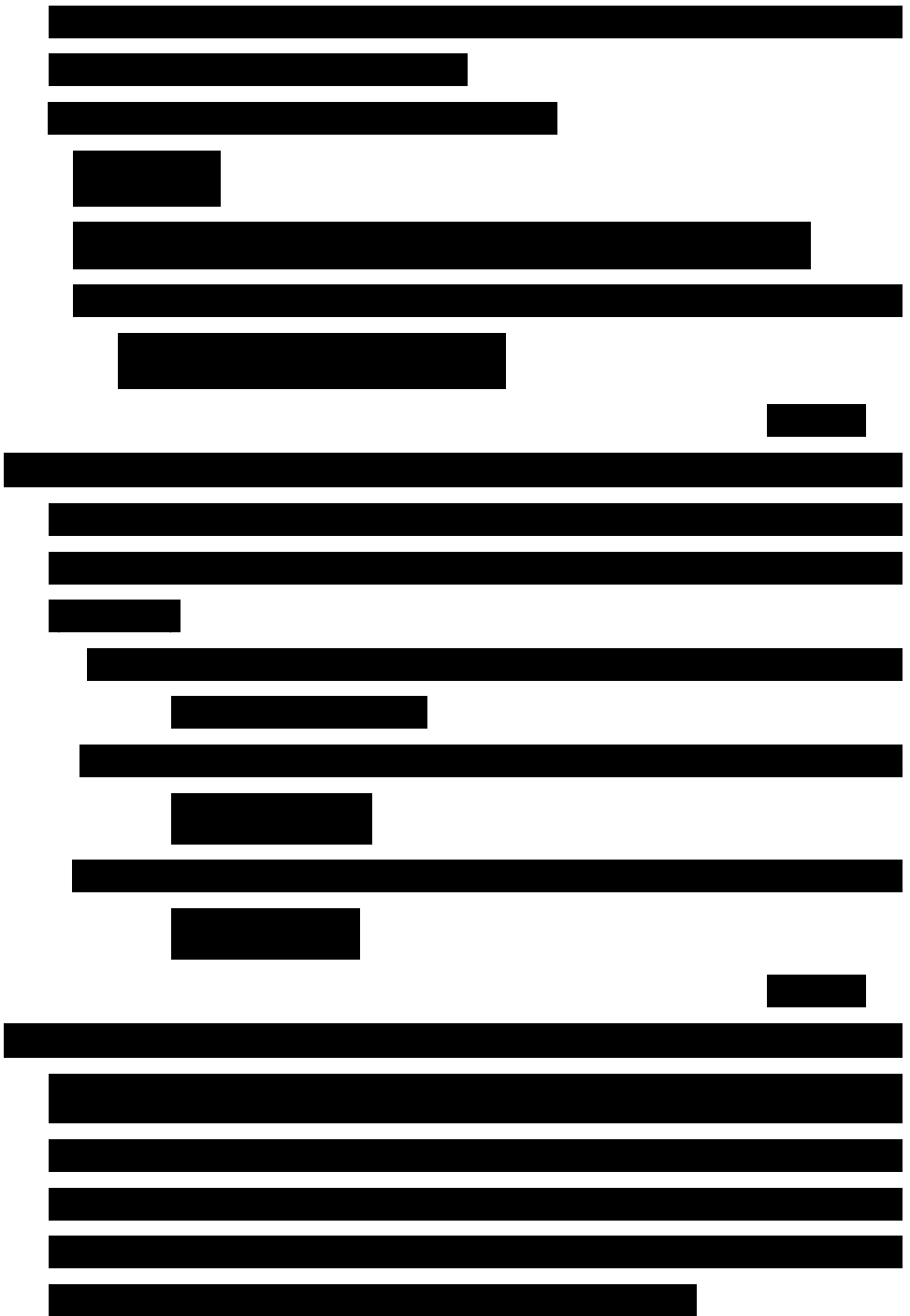
- i. P relative to the balloon, during the interval $0 \leq t \leq t_1$ and
- ii. Q relative to P, during the interval $t_1 \leq t \leq t_2$.

Hence, or otherwise, show that $t_2 = \frac{v}{g} + \frac{1}{2} t_1$. Show further that the velocities of Q and P when the two particles meet are $U \pm \frac{1}{2} gt_1$, respectively.

2009 AL



Motion Graphs Past Papers



Motion Graphs Past Papers

217. Two particles P and Q are simultaneously projected vertically upwards with speeds u and $\frac{u}{\sqrt{2}}$ respectively, from two points on a fixed horizontal floor. There is a fixed smooth horizontal ceiling at a height $\frac{u^2}{4g}$ from the floor. The coefficient of restitution between the ceiling and the particle P which strikes it is $\frac{1}{\sqrt{2}}$, and the two particles move upwards and downwards only under gravity.

- a. Find the speed of the particle P just before it strikes the ceiling and the time T_1 up to the instant of collision.

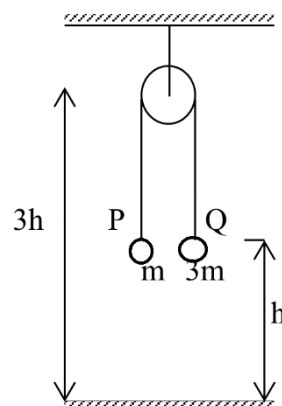
Show that the particle P returns to its point of projection with speed $\frac{u\sqrt{3}}{2}$.

- b. Show that the particle Q just reaches the ceiling and find the time T_2 up to that instant.
- c. Sketch, on the same diagram, the velocity-time graphs for the motions of the two particles P and Q from the instant of projection until they return to the respective points of projection.
- d. Using the velocity time graphs show that, at the instant when P strike the ceiling, Q is at a vertical distance $\frac{u^2}{2g}(\sqrt{2} - 1)^2$ below the ceiling.

2015 AL

Motion Graphs Past Papers

218. A particle P of mass m is connected to a particle Q of mass $3m$ by a light inextensible string passing over a small smooth pulley fixed at a height $3h$ above an inelastic horizontal floor. Initially the two particles are held at a height h above the floor with the string taut, and released from rest. (See the adjoining figure). Applying Newton's second law separately to the motions of P and Q, show that the magnitude of acceleration of each particle is $\frac{g}{2}$.

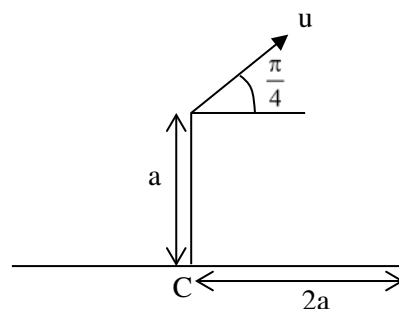


After a time t_0 the particle Q strikes the floor, comes to rest instantly, remains at rest for a further time t_1 , and begins to move up. Sketch the velocity-time graphs separately for the motions of the two particles P and Q until the particle Q begins to move up.

Using these graphs, show that $t_0 = 2\sqrt{\frac{h}{g}}$ and find t_1 , in terms of g and h . Show further that the particle P reaches a maximum height $\frac{5h}{2}$ above the floor.

219. The base of a vertical tower of height a is at the centre C of a circular pond of radius $2a$ on horizontal ground. A small stone is projected from the top of the tower with speed u at an angle $\frac{\pi}{4}$ above the horizontal. (See the figure.). The stone moves freely under gravity and hits the horizontal plane through C at a distance R from C. Show that R is given by the equation gR^2

2016 AL



Motion Graphs Past Papers

$-u^2R - u^2a = 0$. Find R in terms of u , a and g , and deduce that if $u^2 > \frac{4}{3}ga$, then the stone will not fall into the pond.

2017 AI

[Redacted]

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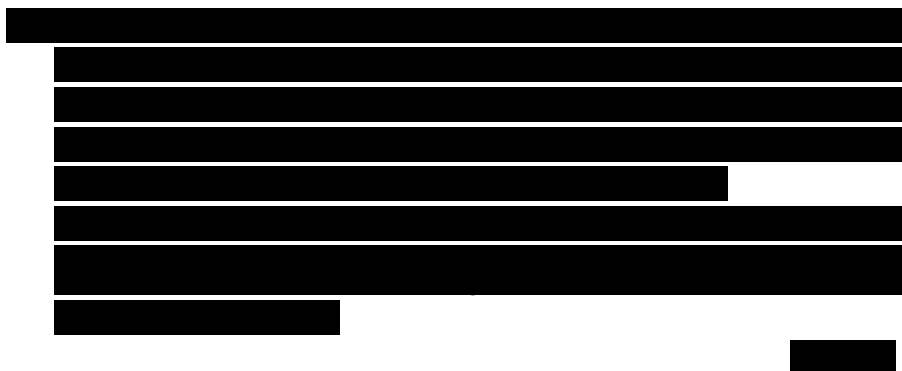
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Collision Past Papers

222. A small smooth sphere A of mass m moving on a smooth horizontal table with velocity u collides directly with another small smooth sphere B of equal size and mass $2m$, which is at rest on the table. The coefficient of restitution is e .

- i. Show that the velocity acquired by B is $(1+e)\frac{u}{3}$, and find the impulse J between the spheres.
- ii. Express the loss of kinetic energy due to the impact in the form $E = \frac{J}{2}(1-e)u$.
- iii. If the direction of motion of A reversed due to the impact, show that $e > \frac{1}{2}$ and $E < \frac{1}{4}mu^2$.

2000 AL



224.

- a. The masses of **three** perfectly elastic smooth spheres A, B, C of equal radii are $\lambda m, m, \lambda m$ respectively where $\lambda > 1$. They are placed on a smooth horizontal plane, with their centers in a straight line, in the above order of the letters. Now B is projected towards A with speed u , so as to strike A directly. Show that,
 - I. the speed of B after this first collision is $\left(\frac{\lambda-1}{\lambda+1}\right)u$.
 - II. B will not strike A again if $\lambda \leq 2 + \sqrt{5}$.
- b. Two masses $3m, m$ are connected by a light inelastic string which passes over a fixed smooth pulley. The system is at rest with the string taut and the portions of the string not in

Collision Past Papers

contact with the pulley vertical, the larger mass being on the ground and the smaller mass hanging freely. A third mass m , falling vertically from rest, through a height h hits the smaller mass and adheres to it, jerking the whole system into motion with speed V . Find the value of V and show that,

I. the impulse set up in the string is $\frac{3}{5}mu$, where $u = \sqrt{2gh}$.

II. the larger mass rises to a maximum height $\frac{h}{5}$, after a time $\frac{u}{g}$ from the instant of the jerk.

2002 AL

225. Two smooth spheres P and Q of small radii and of equal masses are at a point A of a smooth horizontal circular groove of small width and of radius a . At time $t = 0$, the spheres P and Q are projected simultaneously with speeds U and V respectively along the groove in opposite directions, at what time t will the spheres P and Q first collide?

If, after the collision, P and Q move with speeds U_1 , and V_1 , respectively along the groove and if $e (< 1)$ is the coefficient of restitution between the spheres, write down the equations to determine U_1 , and V_1 .

If $U > V$,

- I. Show that, after the collision, Q will move in the direction opposite to its earlier direction of motion.
- II. and if the two spheres move in opposite directions after the collision, show that $e > \frac{U-V}{U+V}$.

If e satisfies the condition given in (ii), show that P and Q will collide again when $t = \frac{2\pi a(1+e)}{e(U+V)}$.

2003 AL

226. A small smooth particle, A and a small smooth elastic particle B of mass m are attached to the two ends of an inelastic string of length l and are at rest on a smooth horizontal plane. The system now moves with speed u in the direction AB with the string taut. The particle B , after some time, collides with a small smooth elastic particle C of mass M which is at rest on the plane. If e is the coefficient of

Collision Past Papers

restitution between the particles B and C , show that the particle B moves with speed $\frac{m-eM}{m+M}u$ after colliding with particle C , and the particle A collides with the particle B after a time $\frac{(m+M)l}{M(1+e)u}$ from the moment of collision between B and C .

2004 AL

[Redacted]

[Redacted]

[Redacted]

[Redacted]

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[Redacted]

230. A simple pendulum of length l hangs at rest with the bob at a height $2l$ above a horizontal floor. A particle of mass equal to that of the bob, strikes the bob horizontally, and subsequently reaches the floor at a point whose horizontal distance from the initial line of the string is $\frac{l}{2}$. If the string turns through an acute angle α before coming instantaneously to rest, show that the coefficient of restitution between the two particles is $\frac{8\sin\frac{\alpha}{2}-1}{8\sin\frac{\alpha}{2}+1}$.

2009 AL

[Redacted]

[Redacted]

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235. A particle P of mass m and a particle Q of mass $3m$ move on a smooth horizontal table along the same straight line towards each other with speed $5u$ and u respectively, as shown in the figure. After their impact, P and Q move away from each other with speeds u and v respectively. Find v in terms of u , and show that the coefficient of restitution between P and Q is $\frac{1}{3}$.



2016 AL

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DYNAMICS

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