A/L Combined Maths

II

DYNAMICS

PROJECTILES & CIRCULAR MOTION

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Projectiles

[redacted]

7. A particle is projected horizontally θ inclined at O velocity of $u\ ms^{-1}$ from point O. After some time t, the particle is in P. The ascending angle from O to P is α.

   i. Show that the projection velocity of the particle is $\frac{gt\cos\alpha}{2\sin(\theta-\alpha)}$.

   ii. Show that if the projection angle is 2α and the ascending angle of P is α, then the projection velocity is $\frac{gt}{2}\cot\alpha$.

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10. When t=0 a particle P is projected horizontally θ inclined from point O at a velocity u ms^{-1}. If the maximum height attained by the particle from O is a m, show that $a = u^2 \sin\theta / 2g$. Let A and B be two points with a vertical height of b m (<a) from the horizontal plane through O. At points A and B, the direction of motion of the particle is inclined α horizontally. Show that $\tan^2\alpha = \left(\frac{a-b}{a}\right) \tan^2 \theta$.

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15. [redacted].

16. When $t = 0$ an object is projected at a velocity v inclined α to the horizontal from the point O, where the maximum height is a and the horizontal range is R, Show that $a = \frac{v^2 \sin^2 \alpha}{2g}$ and $R = \frac{v^2 \sin 2\alpha}{g}$. If the maximum horizontal range is b, obtain that $16a^2 - 8ba + R^2 = 0$. If $b = 25\sqrt{3}$ and $R = 20$ and if the two values for the maximum height are a_1, a_2, show that $a_1 + a_2 = \frac{25\sqrt{3}}{2}$.

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18. Find the flight time of an object to reach a point above 49meters which is projected horizontally inclined 60^0 at velocity 196 ms^{-1} . g = 9.8 ms^{-2}.

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[redacted] A particle P is projected under gravity at an initial velocity u from a point O on the ground at an angle of $\alpha_1 \left(< \frac{\pi}{2}\right)$ to the horizontal. If there is a P particle at a height of h $\left[\leq \frac{u^2}{2g}\ sin^2\ \alpha_1\right]$ above the ground at horizontal distances d_1 and d_2 from O, show that $d_1 + d_2\ = \frac{u^2}{g} \sin 2\alpha_1$ and $d_1 d_2\ = \frac{2hu^2}{g} cos^2\alpha_1$. Deduce that the particle reaches its maximum height after passing a horizontal distance from O to A, OA $= \frac{u^2}{g}\ sin2\alpha_1$. Another Q particle is projected from O [redacted]

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23. When $t = 0$ a particle with an initial velocity $\sqrt{ahg}$ is projected horizontally α inclined from the point O. The particle passes 2 walls at a distance of 2h from each other and a height h. Show that the distance from the projection point to the nearest wall is $h(asin\alpha\, cos\, \alpha - 1)$ and that the equation $sec^4\alpha - (a^2 - 2a)(sec^2\alpha + a^2) = 0$ satisfies fro α. When a = 4, find the projection angle. Then, show that the maximum height at which the particle arrives is $\frac{3h}{2}$.

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26. A bomb explodes vertically h m above the point O on a flat ground. Pieces of the bomb are thrown in all directions at the same speed $\sqrt{2gk}$. If d < 2 $\sqrt{k\,(k+h)}$, show bombs hit twice from different directions at $t_{1,}\, t_{2\,(t_2\, >\, t_1)}$ different times after an explosion on a small animal on Earth at a distance d from point O. Also show that, $t_2 - t_1 = 2\left[\frac{2k+h-\sqrt{h^2+d^2}}{g}\right]^{\frac{1}{2}}$.

27. A particle is projected at a velocity v$\left(> \sqrt{gd}\,\right)$ with an inclination of α to the horizontal from point A on the ground. The particle travels unobtrusively to the top of h pole at a distance of d from A. Show that, h $=$ d $tan\,\alpha - \frac{gd^2}{2v^2}\,(1 + \,tan^2\,\alpha)$. Indicate that h is the highest for the variable value of α when $tan\,\alpha = \frac{v^2}{gd}$ and also get the highest value for h. Show that the maximum altitude at which the particle rises to this value of $tan\,\alpha$ in total flight is $\frac{v^6}{2g\,(v^4 + g^2 d^2)}$.

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33. The maximum height reached by a particle projected with an inclination to the horizontal at a velocity v from point O is h. Let R be the range on a horizontal plane through O. Show that $16gh^2 - 8v^2h + gR^2 = 0$. Find the requirement for the maximum height to be real and depict it by diagram. Hence deduce the maximum horizontal range of the motion.

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38. When a basketball player tries to score a basket, the ball is projected horizontally at angle of α at a velocity of V ms^{-1} . Basket is located at a distance of 4a m and a m height from the player's hand. If the ball falls into a basket, show that the equation of motion of the ball 8ag $tan^2 \alpha - 4v^2 \, tan\alpha + v^2 + 8ga = 0$. When a = 1 m, , v = 10 ms^{-1} and g = 10 ms^{-2} find the two directions in which the ball can be projected and illustrate that moment with an image. Which of the two projection directions is the most appropriate angle for the player to choose? Give reasons.

[redacted]

40. A warship sails forward at V velocity. There is a gun mounted on the ship making an ascending angle α. If the velocity of the bullet is u (>v) relative to the gun, indicate that the range of the bullet is $\frac{2u}{g} sin\alpha\ (ucos\alpha - v)$. Show that the maximum range is obtained when the ascending angle is $cos^{-1}\left[\frac{v+\sqrt{v^2+8u^2}}{4u}\right]$.

41. A bird flies in the sky at a u speed along a straight path inclined at an angle α to the horizontal. When the bird is at point A in its trajectory, a shot is fired at a velocity V inclined at an angle θ to the horizontal from point B directly h distance below A. If the shot hits the bird, show that,

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Projectiles

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47. Show that the trajectory equation of an object projected horizontally α inclined at a velocity V along the horizontal and vertical Ox and Oy axes through O is $y = x \tan\alpha - \frac{gx^2}{2v^2}(1 + tan^2\,\alpha)$.

Two particles are projected at a velocity $\sqrt{5ag}$ from point O in a horizontal plane. The two particles travel along two loci through point A, which is at a horizontal a distance from O and vertical height is a. Show that the horizontal distance between the point where the two particles fall in the horizontal plane through O is a $\sqrt{14}$.

48. A smooth sphere is projected at an angle α to the horizontal point O at a 'a' distance from a vertical smooth wall at a velocity u. The ball hits the wall and returns to point O again. The coefficient of restitution between the ball and the wall is e. Show that $eu^2 \sin 2\alpha = ag(1 + e)$. Deduce that $e \geq \frac{ag}{u^2 - ag}$. Also find the height to the point where the ball hits the wall.

49. Find the horizontal range at the moment of the n^{th} collision when a smooth sphere projected at a velocity V horizontally α inclined from point O on the horizontal plane. Thereby deduce the horizontal range from the moment the sphere slides along the plane. coefficient of restitution is e.

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[redacted] An object is projected downward at a velocity of 30 ms^{-1} on an inclined plane which is 30^0 angular to the horizontal from point O. The displacement of the

[redacted]

52. A smooth sphere moves under gravity from a vertical height h from a point O on a plane which is α angular to the horizontal an collide with it. The coefficient of restitution is $\frac{1}{3}$. If the point of the third collision on the inclined plane is B, show that OB $= \frac{208}{81}\ h\ sin\alpha$.

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56. [redacted]

$\sqrt{3}\ ms^{-1}$ from point O when t = 0, find,

i. Maximum height.
ii. Find the time to reach the maximum height.
iii. Draw a velocity - time graph for the vertical motion of the projection and verify the answers in (i) and (ii) above. g = $10\ ms^{-2}$.

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60. The horizontal and vertical velocity components of the projection of a sphere are v and 2v ms^{-1} respectively. Find the,

    i. Flight time.

    ii. Range on the horizontal plane.

    iii. If the horizontal range is 400 m, find the value of v. $g = 10\ ms^{-2}$.

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Projectiles

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69. Show that the time taken for an object projected at a velocity v when t=0 from a point O inlined α to the horizontal is $\frac{vsinsin\ \alpha}{g}$. When t > $\frac{vsinsin\ \alpha}{g}$ where t = t object is at P. At P the direction of motion is inclined β downwards to the horizontal. If the angle of ascent of point O from point P is θ , show that 2 tan θ = tan α - tan β.

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Projectiles

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[redacted] A golf player strikes a ball from point O as it projects with a velocity of 28 ms^{-1} and an inclination of θ to the horizontal. The ball goes slightly [redacted]

79. An object is projected horizontally θ inclined at a velocity v ms^{-1} from a point O vertically h m above the horizontal ground. The object strikes a point P on the horizontal ground at a horizontal distance R from the projection point. Show that $gR^2tan^2\theta - 2v^2\, R\, tan\theta + gR^2 - 2v^2h = 0$. When $v^2 >$

$g\left[\sqrt{h^2+R^2}-h\right]$, show that there are two loci from O to P. When $v^2 = g\left[\sqrt{h^2+R^2}-h\right]$, show that there is only one locus from O to P.

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85. A particle was projected horizontally at an angle of α from point O from the initial velocity u. Show that the trajectory equation through O relative to the horizontal and vertical axes is $y = x\tan\alpha - \frac{gx^2}{2u^2}(1 + tan^2\alpha)$. Then show that the range on the horizontal plane is $R = \frac{u^2 sin\ 2\ \alpha}{g}$. If $R < \frac{u^2}{g}$, show that the particle has two realistic projection angles $\alpha\ and\ \beta$ ($\alpha < \beta$) that give the R range. If the flight times for those two trajectories are $t_1\ and\ t_{2,}$ show that $t_1 t_2 = \frac{2R}{g}$. Show that t_2 - t_1 $\frac{\sqrt{2}\,u}{g}$ when $\alpha = 15^0$.

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[redacted]

89. An airplane flies directly above the position of a gun at a constant height h and a constant velocity u. What is the minimum exit velocity at which a bullet should be fired at the moment when the aircraft is flying directly above the position of the gun as such the bullet hits the airplane? What is the appropriate ascending angle for this?

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91. A man at the top of a mountain throws a stone at a velocity u in the direction of making an α angle with the upward vertical. After a T interval he throws another stone from that point with a velocity V in the same plane where the first stone moves in the direction of making an angle $(\alpha + \frac{\pi}{2} + \theta)$ with the upward vertical. If the two stones collide, show that when usin $\alpha < v\cos(\theta + \alpha)$, $\mathrm{T} = \frac{2uv\cos\theta}{g[v\cos(\theta+\alpha)+u\sin\alpha]}$.

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[redacted]

94. A shell shot at velocity u inclined 60^0 to the horizontal from point O hits a target at point A in the same horizontal plane. A second shot is fired in the same direction with a velocity v from O. At that moment, the target begins to move vertically at a constant velocity u. The second shot hits the target. Show that $\sqrt{3}\, v^2 - 2uv - \sqrt{3}\, u^2 = 0$. Hence, deduce v = u $\sqrt{3}$.

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96. Taking the initial point of a gun as O, the cooridnates of the bullet P at t time in relation to the horizontal and vertical axes Ox and Oy is A $\equiv (a, b)$. The projection velocity of the bullet is $\sqrt{2gc}$. The projection angle of the bullet is α. Show that the locus of P that satisfies α is $a^2 tan^2\alpha - 4ac\, tan\alpha + a^2 + 4bc = 0$. Show that if $a^2 > 4c(c - b)$, the bullet does not pass-through A, then if $a^2 < 4c(c - b)$, the bullet passes through A in two directions α_1 *and* α_2, then show that tan $(\alpha_1 + \alpha_2) = \frac{-a}{b}$.

97. The horizontal and vertical components of the projection velocity of an object projected under gravity from O relative to the axes Ox and Oy are u and v respectively. Show that the trajectory equation is $2u^2 y = 2uvx - gx^2$. Show

that the horizontal range of the object is $\mathrm{R} = \frac{2\mathrm{uv}}{\mathrm{g}}$. A particle is projected from a point O in a horizontal plane as such it just goes over a wall of $\frac{a}{2}$ height and horizontal distance a away from O. The horizontal range of the particle is 4a. Show that the projection angle of the particle is $tan^{-1}\left(\frac{2}{3}\right)$ inclined to the horizontal and the projection velocity is $\sqrt{\frac{13ag}{3}}$.

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100. A particle of mass mkg is on a smooth horizontal plane. One end of a 2m light inelastic string is attached to the particle and the other end of the string is attached to a point O on the plane. The particle moves on the plane at an angular velocity of 4rads^{-1}. Find the tension of the string. When m = 3kg and the max tension of the string is 150N, find the maximum value of the angular velocity.

[redacted]

102. Two particles of masses m, M kg are attached to the ends of a light inextensible string of length 3am. The string is passed around a smooth peg at the point O. The particle m rotates at an angular velocity of ω rads^{-1} horizontally. The mass M is at the center C of the circle where OC = am. Show that 2m = M and T = $2a$ mω^2.

[redacted]

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105. A particle of mass m kg is attached to a m length two light inelastic strings and the ends of the strings are attached in the same vertical line making α to vertical. The particle moves in a horizontal circle at an angular velocity ω. When the two strings are tight, show that $\omega > \sqrt{\frac{g}{a\cos\alpha}}$. When $\alpha = 60°$, find the minimum value of ω. When $\omega = 4\sqrt{\frac{2g}{a}}$, find the tension, of the strings.

[redacted]

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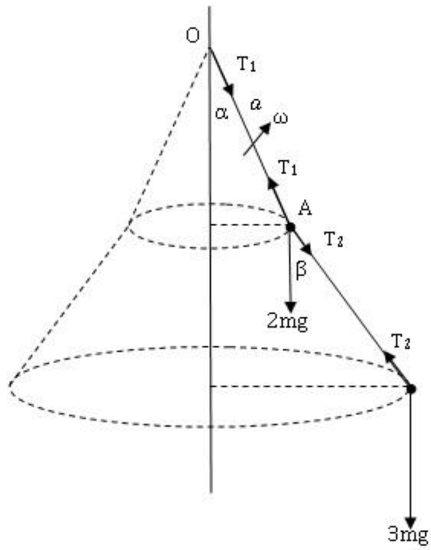
109. A light inextensible string of length am is attached to a point O. A particle A of 2mkg is attached to the other end of the string. Another string of length am is attached to the particle A and the other end of that string is attached to a particle of mass 3m kg. The system with the two strings rotates around the vertical line through O at an angular velocity of $\sqrt{\frac{2g}{a}}$. Show that 5tan $\alpha = 2\ (5\sin\alpha + 3\sin\beta)$ and $\tan\beta = 2\ (\sin\alpha + \sin\beta)$. If $\tan\beta = \frac{12}{5}$, prove that $\sin\alpha = \frac{18}{65}$.



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111. A train compartment moves in a circular railway track of radius am at a velocity of V ms^{-1}. The gap between railway track is 2m. The height to the center of gravity of the train is 1m. Find the vertical reaction on the railway track and find the force on the outside track.

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[redacted]

118. A circular plate of radius a rotate around the center O at an angular velocity of ω in a horizontal plane. The plane surface inside a circle of radius b (a>b) is rough. The coefficient of friction of the rough part is μ. Two particles P_1, P_2 of mass

m_1, m_2 are connected by an inelastic sting and kept in line with the radius of the plate. The particles are kept at rest relative to the plate. The two particles are placed r_1, r_2 distances respectively from the point O. When $r_1 < b < r_2 < a$, the maximum tension that the string can bear is T_o. The two particles are at rest and the both the strings is not slack. Show that $(m_1 r_1 + m_2 r_2)\ \omega^2 \leq \mu m_1 g$ and $m_2 r_2 \omega^2 \leq T_o$. When ω is increased slowly, find how the relative stillness breaks.

[redacted]

122. Two ends of a light inelastic string of 7am is fixed to two points in the same vertical line which are 5am apart. A ring C of mass mkg is passed through the string and the ring rotates in a horizontal circle with an angular velocity of ω

rads^{-1}making $A\hat{C}B = 90°$. If $C\hat{A}B = \theta$, show that $\tan\theta < 1$ and deduce that $AC > CB$. Show that the radius of the horizontal circle is $\frac{12a}{5}$ m. Find the tension of the string and show that the angular velocity of ring is $\frac{1}{2}\sqrt{\frac{35g}{3a}}$rads^{-1}. Since the ring C rotates in a horizontal circle, the vertical acceleration is zero.

[redacted]

126. The plane surface of a solid hemisphere of radius 5m is fixed to a horizontal plane. A particle of mass m kg is placed on the highest point A of the hemisphere. The particle loses its contact with the surface of the hemisphere at the point B and moves in a projectile. The particle hits a vertical wall which is at a horizontal

distance $\frac{6\sqrt{5}\,m}{3}$ from the point O. Find the height from the ground to the point which the particle hits.

[redacted]

128. A particle P is placed at the lowest point A on the smooth internal surface of a hollow sphere of centre O and internal radius a. The particle is projected horizontally giving an initial speed of $\sqrt{nag}$. Here $n > 0$. When the particle is in contact with the sphere and OP turns an angle θ, find the reaction on the particle from the plane. When $2 < n < 5$, show that the speed at which the particle leaves the surface is $\sqrt{\frac{(n-2)ag}{3}}$. If P leaves at a height $\frac{a}{2}$ from the point O, show that,

i) $n = \frac{7}{2}$.

ii) The particle P moves in a path which goes through A under gravity.

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[redacted]

131. A weight A of mass M is attached to a light inelastic string of length a. The weight is on a rough horizontal plane. The other end of the string is attached to a particle B of mass m. The particle m is projected vertically upwards with a velocity $u\left(3 \leq \frac{u^2}{ag} \leq 6\right)$. Assuming the weight M is still, find the tension of the string when the string AB makes an angle θ with the horizontal. Show that the perpendicular reaction on the weight from the horizontal plane is $Mg\left[1 - \frac{m}{M}\sin\theta\left(\frac{u^2}{ag} - 3\sin\theta\right)\right]$. Hence show that the weight A is in contact with the horizontal plane through out the semi circle when $\frac{M}{m} > \frac{1}{12}\left(\frac{u^2}{ag}\right)^2$.

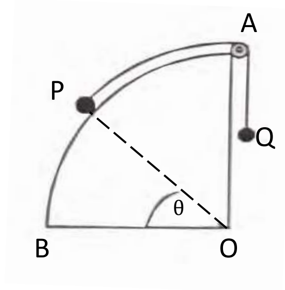
132. Two rings A, B of mass 6m, 4m kg are attached to a light inelastic string of length a. The two rings are free to move along a smooth vertical circular wire of center O. AB makes an angle 60° with the horizontal line through the center of the circle. When t =0, the two rings A, B are released from rest by keeping them in the same horizontal line above the point O. When the string makes an angle θ with the horizontal,

I. Show that $\dot{\theta}^2 = \frac{g}{3a}\left(\sin\theta - 3\sqrt{3}\cos\theta + 3\sqrt{3}\right)$.

II. Show that the tension of the string is $\frac{16\sqrt{3}}{9}$ mg cos θ.

III. Find the reaction on the ring B.

133. The figure shows a vertical cross section of a fixed solid quadrant of radius a. A light inelastic string passes through a pulley A which is at the top of the quadrant. The ends of the string are connected to particles P, Q of masses m and M $(M > m)$ respectively. The motion starts when OP(OB) is horizontal. Show that $(M+m)a\dot{\theta}^2 = 2g(M\theta - m\sin\theta)$. Here θ is the angle between OP and OB at time t. Hence find the reaction on the particle P from the curved surface. Show that,



I. The value of $\cos\alpha = \frac{2M}{M+3m}$ becomes maximum when $\theta = \alpha$,when $M < 3m$.

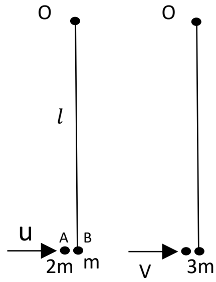
II. The β value which satisfies the equation $\theta = 0$ or $\sin\beta = \frac{2M\beta}{M+3m}$ vanishes when $\theta = \beta$ and $\frac{3m}{M} < \pi - 1$.

[redacted]

135. A particle of mass mkg is attached to a light inextensible string of length 3am. The particle hangs in equilibrium. The particle is projected horizontally by giving a velocity of ums^{-1}. When the string is horizontal, the string hits a small peg at A which is 2a distance from point O. If the circle just completes a full circle, find the minimum speed u of the particle.

[redacted]

[redacted] A particle of mass mkg is attached to a light inextensible string of length l. The other end of the string is attached to a point O and the particle hangs in equilibrium. A particle B of mass 2mkg hits the particle A with a horizontal velocity of u and the two particles [redacted]



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Circular Motion

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152. There is a smooth vertical boundary around a smooth fixed circular plate of radius am. A particle of mass mkg moves at a speed of Vms^{-1} touching the edge of the plate. Find the reaction on the particle from the boundary. If m = 2kg, V = 4ms^{-1} and $a = 2$m, deduce the value of R.

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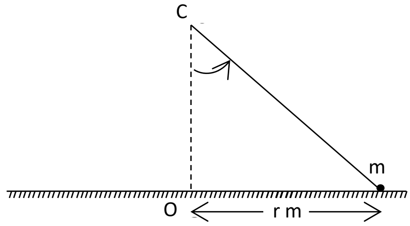

154. A particle of mass mkg is on a rough horizontal plane. The particle is attached to a light inelastic string and the other end of the string is fixed to a point C which is right above the point O. The particle rotates in a horizontal circular path of center O and radius rm at an angular velocity of ω rads^{-1}. The string makes an angle α with the vertical at limiting equilibrium. The gravitational acceleration is gms^{-2} and the coefficient of friction is μ. Show that the tension of the string T is $\frac{mr\omega^2-\mu mg}{\sin\alpha-\mu\cos\alpha}$. If $\omega^2=\mu g$, $r=2$m, $\mu=\frac{1}{3}$ and $\alpha=sin^{-1}\left(\frac{5}{13}\right)$, deduce that $T=\frac{13}{3}$ mg.

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[redacted]

157. A particle P of mass mkg rotates in a horizontal circle at an angular velocity of ω revs^{-1} on the inner surface of a fixed smooth bowl of radius a and center O and having a horizontal edge. Show that the angle made by OP with the downward vertical is $\cos^{-1}\left(\frac{g}{4a\pi^2\omega^2}\right)$. If $\omega=\frac{\sqrt{g}}{\pi}$ and $a=\frac{1}{2}$m, show that $\alpha=60^{\circ}$. Show that the radius of the circle that the particle rotates is $\frac{a\sqrt{3}a}{2}$.

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161. A light inelastic string of length l is passed into a fixed small smooth ring R. The two ends P and Q of the string are attached to the particles of masses m and $\lambda m (\lambda > 1)$ respectively. The particle P moves in a horizontal circle at a constant angular velocity of ω around the center C, where C is a point which is right below the point R. The particle Q is at rest at the point C. Show that,

$$\omega^2 = \frac{g(1+\lambda)}{l}$$

(ii) A force of magnitude $mg\sqrt{2\lambda(1+\lambda)}$ acts on the top of the string.

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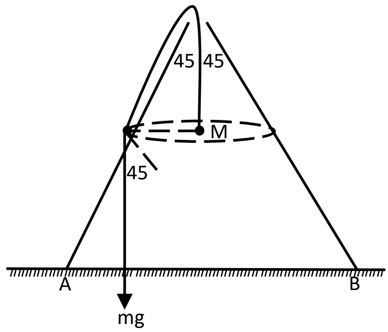

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163. A motor vehicle of mass Mkg moves in a circular road of radius a and angle of inclination α to the horizontal at a velocity of V ms^{-1}. If the vehicle is just about to slip, show that the speed of the vehicle is $\sqrt{\frac{ag(\mu+\tan\alpha)}{1-\mu\tan\alpha}}$. If $\alpha = 30^{\circ}$ and $\mu = \frac{1}{\sqrt{3}}$, deduce that $V = \sqrt{3ag}$. Find the frictional force and the perpendicular reaction at that moment. μ is the coefficient of friction.

164. A motor vehicle of mass Mkg moves in a circular road of radius a and angle of inclination α to the horizontal. When the speed of the vehicle is Vms^{-1}, the vehicle tends to move downward. Show that $V_{min} = \sqrt{\frac{ag(\sin\alpha-\mu\cos\alpha)}{\mu\sin\alpha+\cos\alpha}}$. Let $a = 10$m, $g = 10$ms^{-2} and $\alpha = 45^{\circ}$. If $V_{min} = 5$ms^{-1}, show that $\mu = \frac{3}{5}$.

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169. Point A is h distance above the point O where the point O is on a smooth horizontal plane. One end of a light inelastic string of length l is attached to A and the other end of the string is connected to mass m. The mass m is kept on the horizontal plane. The string is just tight. The m particle is projected perpendicular to the string in order to move it in a horizontal circle of center O at an angular velocity of ω.

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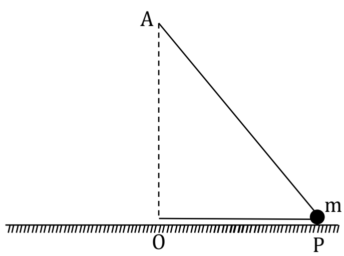

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Circular Motion

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181. [redacted]

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182. A particle of mass m is projected horizontally at the lowest point A inside a smooth hollow cylinder of radius a with a velocity of u. If $0 < u < \sqrt{2ag}$, show that the particle A, oscillates at an angle $2\cos^{-1}\left(\frac{2ag-u^2}{2ag}\right)$. If $u = \sqrt{ag}$, deduce that the particle oscillates 120° around A. Show that the reaction at the highest point of the motion is $\frac{mg}{2}$.

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195. A particle P of mass mkg is attached to a light inelastic string of length lm. The other end of the string is fixed to a point O and the particle hangs at rest. The particle is projected at a horizontal velocity of $(lg)^{\frac{1}{2}}$ ms^{-1}. If the particle P moves in a full circle around the center O, show that $k \geq 5$. When the particle is at horizontal position through O, find the tension of the string.

[redacted] A particle P of mass mkg is attached to a light inelastic string of length am. The [redacted]

[redacted]

199. A smooth circular wire of radius a is fixed in a vertical plane. Two rings of masses m_1 and m_2 which are smoothly hinged at the two ends of a light rod, can move freely along the circular wire. Show that the stress in the rod is $\frac{2m_1m_2}{m_1+m_2}\, g \tan\alpha \cos\theta$. Here the angle subtended at the center from the rod is 2α and the angle of inclination of the rod with the horizontal is θ. Write an expression for reaction between the ring and the wire in terms of α and $\dot{\theta}$.

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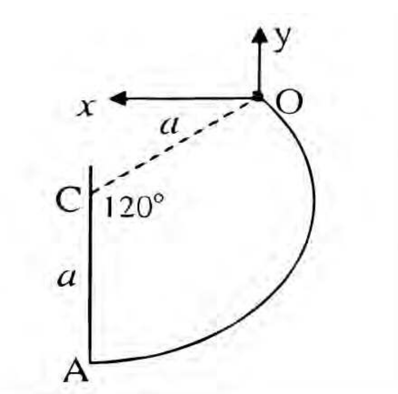

Circular Motion

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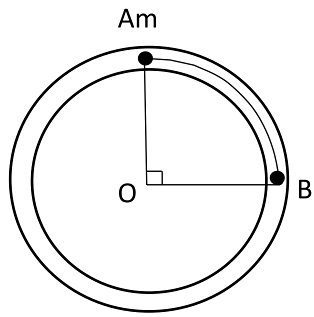

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206. A particle P is projected under gravity with initial speed U at an angle. $\alpha_1 \left(< \frac{\pi}{2}\right)$ with the horizontal from a point O on the ground. If *P* is at a height $\mathrm{h}\left(\leq \frac{\mathrm{U}^2}{2\mathrm{g}} \sin^2\alpha_1\right)$ above the ground when it is at horizontal distances d_1 and d_2 from O, show that $\mathrm{d}_1 + \mathrm{d}_2 = \frac{\mathrm{U}^2}{\mathrm{g}} \sin 2\alpha_1$ and $\mathrm{d}_1\mathrm{d}_2 = \frac{2\mathrm{hU}^2}{\mathrm{g}} \cos^2\alpha_1$.

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209. A smooth particle P is projected with velocity u at an angle $\alpha, \left(0 < \alpha < \frac{\pi}{2}\right)$ to the horizontal, under gravity.

The particle P, at the instance it moves horizontally, strikes another smooth particle Q of equal mass at rest hanging from one end of an inextensible string of length l, the other end of the string being attached to a point O on a horizontal rail. The rail is perpendicular to the vertical plane in which the path of the particles P and Q lies. Show that, the horizontal distance between the two particles P and Q initially is $\frac{u^2 \sin 2\alpha}{2g}$.

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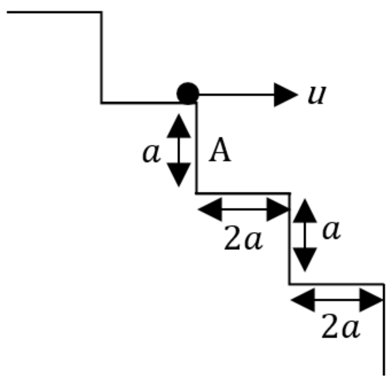
213. Two particles P and Q are placed at a point O on a fixed smooth plane inclined at an angle α $\left(0 < \alpha < \frac{\pi}{2}\right)$ to the horizontal. The particle P is 2 given a velocity u upwards along the line of greatest slope through O, and at the same instant the

particle Q is released from rest. Assuming that the two particles do not leave the inclined plane, sketch the velocity-time graphs for the motions of P and Q on the same diagram.

Using these graphs, show that, at the instant the particle P returns to the point O, the particle Q is at a distance $\frac{2u^2}{g\sin\alpha}$ from O.

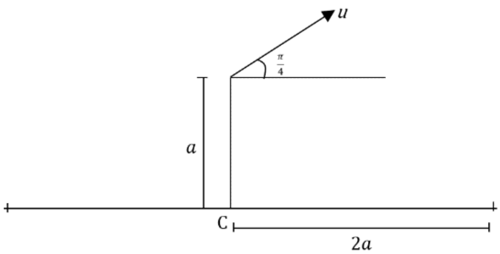
2014 A/L

214. A particle P, projected horizontally with velocity u given by $u\ \frac{3}{2}\sqrt{ga}$ from a point A at the edge of a step of a fixed stairway perpendicular to that edge, moves under gravity. Each step is of height a and length $2a$ (see the figure). Show that the particle P will not hit the first step below A, and it will hit the second step below A at a horizontal distance $3a$ from A



2016 A/L

215. The base of a vertical tower of height a is at the centre C of a circular pond of radius $2a$, on horizontal ground. A small stone is projected from the top of the tower with speed u at an angle $\frac{\pi}{4}$ above the horizontal. (See the figure.) The stone moves freely under gravity and hits the horizontal plane through C at a distance R from C. Show that R is given by the equation $gR^2 - u^2 R - u^2 a = 0$.



Find R in terms of u, a and g, and deduce that if $u^2 > \frac{4}{3}\ ga$, then the stone will not fall into the pond.

2017 A/L

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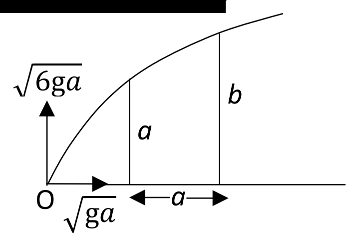

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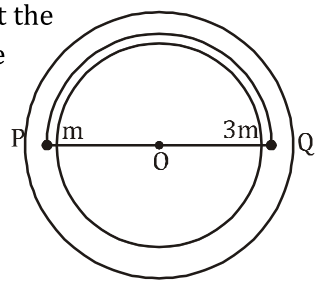
219. The Figure depicts a narrow smooth tube bent in the form of a circle of centre O and radius a, fixed in a vertical plane.

Inside the tube are two particles P, Q of masses m, 3m respectively connected by a light inextensible taut string of length πa. Initially, the system is released from rest, when the particles are at the opposite ends of the horizontal diameter of the tube and the string occupies the upper half of the tube.



If OP has turned through an angle θ, a time t after release, using the principle of conservation of energy show that,

$$a\,\dot{\theta}^2 = g\sin\theta \left(0 \le \theta \le \frac{\pi}{2}\right)$$

Find the force exerted by the tube on the particle P, at this instant.

2001 A/L

220. A particle P is released from rest at a point A on the smooth outer surface of a fixed sphere of centre O and radius a, OA making an acute angle α with the upward vertical. At time t, with P still on this surface, OP makes an angle θ with the upward vertical. Show that,

i) $\dot{\theta}^2 = \frac{2g}{a}(\cos\alpha - \cos\theta)$;

ii) The particle will leave the surface when $\cos\theta = \frac{2}{3}\cos\alpha$

2002 A/L

221. A small smooth particle P of mass m is free to move under gravity in a thin smooth circular tube of radius r and centre O, fixed in a vertical plane. The particle is projected horizontally from the lowest point of the tube with speed $\sqrt{3\,gr}$.

Explain why the law of conservation of energy can be applied for the motion of the particle.

If v is the speed of the particle when OP makes an angle θ with the downward vertical, show that $v^2 = gr(1 + 2\cos\theta)$

Hence show that the reaction of the tube on the particle changes its direction when $\theta = \cos^{-1}\left(-\frac{1}{3}\right)$ and find the speed of the particle at that point.

2004 A/L

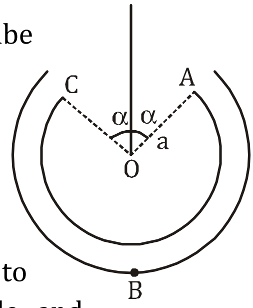
222. A particle P of mass m is attached to a fixed-point O by a light inextensible string. The particle, held with the string taut and making an angle $\alpha \left(< \frac{\pi}{2}\right)$ with the downward vertical, is given a velocity u, perpendicular to the string, in the vertical plane through OP. Assuming that the particle is in circular motion, write down the equation of conservation of energy for the particle, by considering the general position where OP makes an angle θ with the downward vertical.

Show that the particle describes a circular arc until OP makes an angle

$\cos^{-1}\left[\frac{1}{3}\left(2\cos\alpha - \frac{u^2}{ga}\right)\right]$ with the downward vertical and then begins to move freely under gravity, provided that $ga(3 + 2\cos\alpha) > u^2 > 2ga\cos\alpha$.

2005 A/L

223. a) The figure given below depicts a smooth narrow tube ABC bent into the form of a circular arc of centre O, radius a and angle 2 $(\pi - \alpha)$, where α is an acute angle. The tube is fixed in a vertical plane with the two open ends A, C uppermost and at the same horizontal level. A particle, placed at the lowest point B of the tube, is projected with a horizontal velocity u. The particle moves through the tube to the end A, then moves freely under gravity as a projectile, and enters the tube again at the other end C. Find the velocity of the particle as it leaves the tube at A, and



show that $u^2 = ga\,[2(1+\cos\alpha) + \sec\alpha]$.

Show further that the greatest height reached by the particle is $\frac{a}{2}$ $(\cos\alpha + \sec\alpha)$ above O.

b) A particle P is released from rest at a point A on the smooth outer surface of a fixed sphere, of centre O and radius a, where OA makes an acute angle α with the upward vertical. Show that, when OP makes an angle θ with the upward vertical, with P still on the surface of the sphere $a\dot{\theta}^2 = 2g$ $(\cos\alpha - \cos\theta)$. Find the value of θ at the point where the particle P leaves the surface.

2006 A/L

224. A smooth sphere with centre O and radius a, is fixed on to the horizontal surface of a table. A smooth particle P is placed on the outer surface of the sphere at a point A where OA makes an acute angle α with the upward vertical. The particle is released from rest ,

i) Show that, when OP makes an angle θ with the upward vertical and the particle is still in contact with the surface of the sphere, $a\dot{\theta}^2 = 2g$ $(\cos\alpha - \cos\theta)$

ii) Find the magnitude of the reaction between the sphere and the particle.

[redacted]

225. A bowl is made up from a fixed smooth spherical shell with centre O and radius a by removing the upper part cut off by the horizontal plane at a distance $\frac{a}{4}$ above O. A particle P with mass m is projected horizontally from the lowest point inside the bowl with speed u.

i) Find the speed of the particle and the magnitude of the reaction between the bowl and the particle when OP makes an angle θ with the upward vertical.

ii) Show that the particle will leave the edge of the bowl, provided that $u^2 > \frac{11\,ga}{4}$

iii) Also show that, in the subsequent free motion under gravity after the particle leaves the bowl, will not fall back into the bowl. Provided that $u^2 > \frac{13\,ga}{2}$

2008 A/L

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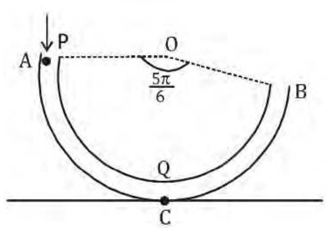
227. A thin smooth tube ACB in the shape of a circular arc of radius a that subtends an angle $\frac{5\pi}{6}$ at its centre O is fixed in a vertical plane with OA horizontal and



the lowest point C of the tube touching a fixed horizontal floor as shown in the figure. A smooth particle P of mass m is projected vertically downwards into the tube at the end A with speed $\sqrt{2ga}$.

Show that the speed of the particle P, when OP makes an angle θ $\left(0 \leq \theta \leq \frac{\pi}{2}\right)$ with OA is $\sqrt{2ga\,(1 + \sin\theta)}$ and the magnitude of the reaction on the particle P from the tube is mg (2 + 3 sin θ). The particle P, when it reaches the point C, strikes another smooth particle Q of mass m which is at rest inside the tube at C. The coefficient of restitution between the particles P and Q is $\frac{1}{2}$.

[redacted]

228. A Particle P of mass m is attached to one end of a light inextensible string of length l and the other end of the string is attached to a fixed-point O. When the particle P is hung freely in a vertical Plane it is given a velocity$\sqrt{2gl}$ in the vertical plane perpendicular to OP. Using the Principle of Conservation of Energy, find the velocity of the particle P when OP makes an angle $\frac{\pi}{3}$ with the downward vertical. Show that the tension of the string at this instant is $\frac{3}{2}$ mg.

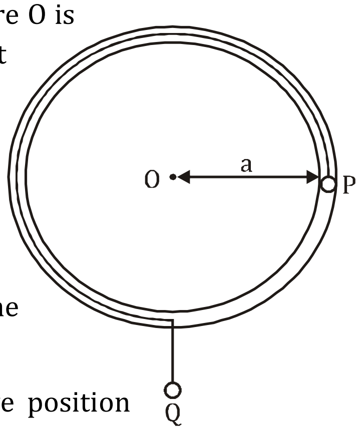
2012 A/L

229. A particle P of mass m is placed at the highest point of the smooth outer surface of a fixed sphere of radius a and centre O. Another particle Q of mass 2m moving horizontally with velocity u collides directly with P. The coefficient of restitution between P and Q is $\frac{1}{2}$. Find the velocity of P just after the collision.

Assuming that the particle P is still in contact with the sphere when the radius OP has turned through an angle θ, show that the magnitude of the reaction on the particle P from the sphere is $\frac{m}{a}\,[ga\,(3\cos\theta - 2) - u^2]$ Also show that if $u = \sqrt{ga}$, then the Particle leaves the surface of the sphere just after collision with Q.

2013 A/L

230. A narrow smooth circular tube of radius a and centre O is fixed in a vertical plane. One end of a light inextensible string of length greater than $\frac{3\pi a}{2}$ is attached to a particle P of mass m which is held inside the tube with OP horizontal. The string passes through the tube and through a small smooth hole at the lowest point of the tube as shown in the figure and carries a particle Q of mass 2m at the other end.



The particle P is released from rest from the above position with the string taut. By applying the Principle of Conservation of Energy, show that the speed v of the particle P when OP has turned through an angle $\theta \left(0 < \theta < \frac{3\pi}{2}\right)$ is given by $v^2 = \frac{2ga}{3}(2\theta - \sin\theta)$ and find the reaction on the particle P from the tube.

2014 A/L

231. A straight narrow smooth tube OA, of length l, with both ends open, is fixed with the upper end O at a height h ($> l$) above the horizontal ground, making an angle $\frac{\pi}{3}$ with the downward vertical. A particle gently placed inside the tube at O slides down along the tube. Next the particle leaves the tube at the end A and strikes the ground at a point B at a horizontal distance $\sqrt{3}l$ from O. Show that (i) the speed of the particle at A is $\sqrt{gl}$ and (ii) $h = 3\,{}^{l}/_{2}$.

2015 A/L

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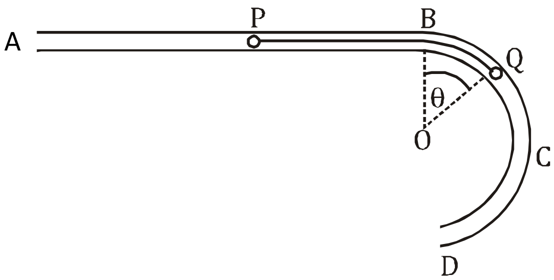

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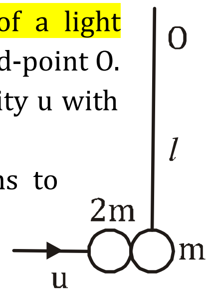
233. A particle of mass m hangs in equilibrium at one end of a light inextensible string of length l whose other end is tied to a fixed-point O. Another particle of mass 2m collides horizontally with velocity u with the first particle and coalesces with it.
Find the velocity with which the composite particle begins to move. Show that if $u = \sqrt{gl}$, then the composite particle reaches a maximum height of $\frac{2l}{9}$ above its initial level.



2016 A/L

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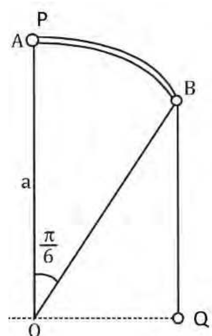

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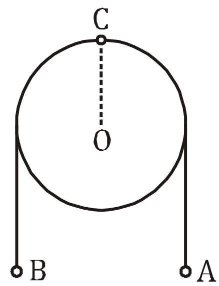
$\theta\left(0 < \theta < \frac{\pi}{6}\right)$ with the upward vertical, show that $2a\dot{\theta}^2 = 3g\,(1 - \cos\theta) + g\theta$ and that the tension in the string is $\frac{3}{4}$ mg (1 - sin θ), and find the normal reaction on the particle P.

2016 A/L

235. Two particles A and B, each of mass m are attached to the two ends of a light inextensible string of length l ($> 2\pi a$). A particle C of mass 2m is attached to the mid-point of the string. The string is placed over a fixed smooth sphere of centre O and radius a with the particle C at the highest point of the sphere, and the particles A and B hanging freely in a vertical plane through O, as shown in the figure.



The particle C is given a small displacement on the sphere in the same vertical plane, so that the particle A moves downwards in a straight-line path. As long as the particle C is in contact with the sphere, show that $\dot{\theta}^2 = \frac{g}{a}(1 - \cos\theta)$, where θ is the angle through which OC had turned. Show further that the particle C leaves the sphere when $\theta = \frac{\pi}{3}$.

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