

A/L Combined Maths

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AWS

PULLEYS & WEDGES

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- 7. A fixed pulley A is hung from point O in the roof. From the two sides of the inextensible string around the pulley, a particle P of mass 2m and a smooth pulley B of mass m are hung. At the the two ends of the inextensible string around the pulley B two particles Q and R of masses m and km respectively are attached. Find the acceleration of the pulley B. If k = 2, find the acceleration of R relative to B and the tension of the strings.
- 8. The inextensible string attached to the m₁ kg particle, the other end is attached to the particle m₂ kg kept on a smooth plane, through a pulley A. The inextensible string attached to m₂ is drawn through the pulley B and attached to a vertically hanging m₃ particle, and the system is released from rest. Find the acceleration of the particles and the tension of the strings. If m₁ = 3mkg, m₂ = 2mkg, m₃ = mkg, show that $a = \frac{g}{3}$ ms⁻² and tension of the string 2mg.



An inextensible string is attached to a fixed-point O from one end and goes below a movable smooth pulley A of mass M_1 and above a fixed pulley B and below a movable pulley C of mass M_2 and the other end is attached to the pulley A. Find the acceleration of A and C, when the system is released from rest. Obtain the tension of the string and if $2M_2 > M_1$, show that A move downwards.



13. An inextensible string attached to a particle m_1 on a rough horizontal plane goes around fixed smooth pulley A at the edge of the plane and below a smooth moveable pulley of mass M and above a fixed pulley B and at the other end it is attached to a particle m_2 on a rough plane. Find the acceleration of the particles and the tension of the strings when the system is released from rest. Let coefficient of friction of m1 and m2 be μ_1 and μ_2 respectively.



If $m_1 = 2m$, $m_2 = m$ and $\mu_1 = \mu_2 = \frac{1}{2}$, find the acceleration of the particles and show that the requirement for motion is 5M > 8m.





of friction be μ . Find the acceleration of the particles and the tension of the strings. If k = 2, deduce that for motion it is required that $\mu < \frac{1}{3}$. If $\mu > \frac{1}{3}$, find the acceleration of B and then find the velocity of B after 1 second.





20. A particle P of mass m kg is on a AB rough plane of coefficient of friction $\frac{1}{\sqrt{3}}$ and 30° inclined to the horizontal, where AB = 2a, and attached to an inextensible string of length a such that the other end of the string is connected to a particle Q of mass M kg, through a smooth pulley A. The



particle Q is kept close to A and released slowly. Show that the common acceleration of P and Q is $\left(\frac{M-m}{m+M}\right)g$. The particle Q hits on the inextensible floor and comes to rest. Find the horizontal distance from A to the position where the particle P hits the floor and the time for that action. Take the common acceleration of the particles as a_1 .

21. An inextensible string is attached to a particle P of mass m on a rough plane with coefficient of friction $\frac{1}{\sqrt{2}}$ inclined 30° to the horizontal and goes around a smooth fixed pulley A at the top of the inclined plane and goes below a smooth moveable pulley B of mass M and goes around smooth fixed pulley C which is l_1 distance above B and goes below the pulley D at the top of the smooth plane 60° inclined to the horizontal and is attached to the particle Q of mass 2m on the inclined plane. t time after



releasing the system from rest, if AP = x and DQ = y, find vertical distance from B to C. Hence, find the acceleration of P and Q in terms of x and y, and find the acceleration of B, if B moves downwards. Show that the tension of the string is $\frac{(6 + \sqrt{3}) \text{ mMg}}{(8m+3M)}$. Obtain the acceleration of P and Q.





26. A M kg mass ABC wedge is kept on a smooth horizontal plane keeping A above Bc and two particles of mass mkg and m' kg is kept on the sides of the wedge with the inclinations α and α' respectively, and the system is released from rest. Find the acceleration fo the wedge. If $\alpha = \alpha' = 30^\circ$, AB = l and m > m', and the particles are released from A, find the displacement of the wedge when the particle m reaches B.





28. A square cross section of a wooden log of mass M is kept on a smooth horizontal plane. An inextensible string is attached to a point A from one end, and it goes through a smooth pulley B on the top of the wedge, and from the other end it is attached to a particle of mass m on the α



inclined plane of the wedge on a smooth horizontal plane, and the system is mass of the wedge is m₁ released from rest. Write necessary equations to find the acceleration of the parts in motion and the tension of the string. If $M = m_1 = m$, find the acceleration and the tension. Find the requirement for motion and show that the reaction between the wedge and the particle is $\left(\frac{\cos^2 \alpha - 4\cos \alpha + 1}{\cos^2 \alpha + 2\cos \alpha - 5}\right)$ mg.

29. The diagram illustrates a system of weights and pulleys. A right circular cone of mass M and A,B,C semi vertical angle α . It is attached to a string which goes above a fixed smooth pulley R and below a smooth pulley S. The slack string is attached to the fixed-point O. The parts of the string where it is not in contact with the pulleys are vertical. Two particles of unit mass m are symmetrically kept on the smooth surfaces of the cone. Show that the cone goes down with an acceleration of $\frac{4\text{mgsin}^2\alpha}{3M+4\text{msin}^2\alpha}$. Also, find the tension of the string. Mass of pulle S is 2M



Pulleys & Wedges



33. The diagram illustrates a smooth cross-section APQR of a wooden log such that it takes the shape of a rectangle. Let the mass of kg be M. Let $RQ = \sqrt{3} a$ and PB = BQ = a. The particle of mass m is made to move freely in the AB small smooth tunnel. If M = 2m, obtain the speed with which the particle leaves



the tunnel. Also obtain the speed of the wooden log. Find the velocity at which the mass m hits the floor.

34. The cross section of a smooth wedge of mass M is a ABC triangle and kept on a smooth horizontal table scuh that the side AB in contact with the table. In the plane of $BÂC = \alpha$ and a hole with a small diameter is carved on the BC side parallel to AB. A particle of mass m is kept inside the hole and attached to an inextensible string going through



the vertex C and attached to the particle of m' on the AC side, and the whole system is slowly released from rest. If $m > m' \cos \alpha$, prove that the wedge moves towards BA according to the order of letters. Find the acceleration of mass m.



Pulleys & Wedges



Two particles X and Y of masses 2m and m kg respectively are kept on the rough inclined planes PA and PB respectively such that PA = PB = 2a and joined by the two ends of an inextensible string going through the smooth pulley P. In the beginning when the string is stretched, the distance from P to X is a m. Find the common acceleration of the particles and the tension of the string. Take coefficient of friction as M

37.







45.

One end of an inextensible string is attached to M kg mass stage at L, and the goes through a smooth pulley A and is attached to a smooth pulley B of mass mkg. One end of an inextensible string is attached to the stage at N, and the goes through a smooth pulley B and is grabbed by a man of mass 2mkg.

- i. If the man exerts a TN force to the string, when the stage is in equilibrium, show that $T = \frac{(m+M)g}{2}$.
- ii. When the stage is in equilibrium, show that the maximum mass of the stage is 7m.
- iii. If the distance from the stage to B is hm and M > 7m, show that the

time taken by the man P to reach B is $t = \sqrt{\left(\frac{M+19m}{M-7m}\right)}$

46. αCross-section of a gate of a palace is illustrated in the diagram. It consists of two symmetrical parts of mass Mkg and mkg. Let the space available to enter when the gate is opened is 2*l*m. Show that when the system is released, the time taken for the gates



to close is $2l\sqrt{\frac{M+2m(1-\cos\alpha)}{mg\sin\alpha}}$. Show that the tension of the string on the top of the gate is $\{M + m(1-\cos\alpha)\}[\frac{mg\sin\alpha}{M+2m(1-\cos\alpha)}]$.



50. A ABC smooth wedge of mass M is kept on a smooth horizontal plane. A particle P of mass kmkg is projected upwards with a velocity of usin α from the point A on the foot of the side AB inclined α to the horizontal. Show that the time taken for the particle to return to the point A is $\frac{2u(M+kmsin^2\alpha)}{(M+km)g}$. Show the displacement of the wedge during that time.





59. At a place where a bridge is being constructed where a pile driver action is being underwent a big hammer of mass metric ton M is being released on to a pillar of mass metric ton m from a height of hm. If after the collision of the big hammer, it doesn't bounce back, find the velocity of the pillar and the big hammer. Resistance to the motion of the pillar is constant and the resistance is R metric ton. If the pillar and the big hammer goes a distance of $\frac{h}{2}$ m into the ground, show that $R = \frac{2M^2 + (m+M)^2}{m+M}$.



62.

- i. When a force P is exerted onto an object of mass 60kg, the objects gains a velocity of 90kmh⁻¹ within a minute. Find P. Obtain the displacement of the object.
- ii. The mass of a train engine and train cabine is 100 ton. The engine give a forward force (Tractive force) of 2 ton. The resistive force for 1 ton is 50 N.
 a) Find the acceleration and deceleration and the distance travelled under retardation.
 b) Also, obtain the total distance travelled.
- 63. i. An object of mass 500kg is dragged along a plane inclined 30° to the horizontal with a force of 15KN. The coefficcient of friction between the object and the plane is $\frac{1}{2\sqrt{3}}$. Find the time taken for the object to move along the 75m inclined plane and the velocity obtained.
 - ii. A train of mass 200 ton moves along a flat railway track with a velocity of 36kmh⁻¹. The driving force (tractive force) is constant, and the magnitude is 75KN. The resistive force for 1 ton is 25N. Find the time taken for the velocity of the train to become 90kmh⁻¹. Obtain the displacement during that time.





66.

- i. A lift of a mine is lifted by a rope. The mass of the lift and the man is Mkg. The lift started from rest obtains a velocity of 4ms⁻¹ within a distance of 6m.
 - a) Find the acceleration of the upwards motion and the tension of the string.
 - b) If M = 300kg, deduce the value of the tension. Let g = 10ms⁻². If the mass of the man is mkg, find the reaction acting on the feet.
- ii. Five concrete cubes of mass mkg is taken to a building of 10m in height using a lift. It is rised up to a 6m height with an acceleration of $\frac{1}{2}$ ms⁻² from rest and after that it travels in uniform decelaration and comes to rest at the top. If cubes are kept on each other, find the reaction on each instance. If cubes were kept on top of each other, find the reaction during each instance. Explain why the reactions change. Obtain the tension of the string if M = 50kg and the mass of the lift is 20kg. Let g = 10ms⁻².





A worker B, who is on <mark>the first floor</mark> of a building to paint, hangs a bucket of mass Mkg with a rope going around a fixed pulley A.

69.

- i. The bucket comes to the ground from A within 2s. Find the tension of the rope.
- ii. 3mkg mass of paint is filled at ground and is pulled the bucket to the upper floor within 4s. Find the force applied by the



worker to drag the bucket. If M = 10, what is the force.

iii. Find the reaction between the bucket and the paint. Explain the instance. Let $g = 10 \text{ms}^{-2}$.







- 73. A lift in a building during its motion of moving down from the top, the first one third it is at rest and comes down with a constant acceleration. In the next one third it goes down with constant velocity. In the last one third it moves with constant deceleration as such it comes to rest as it reaches foot of the building. It is seen that time taken to descend is as much as time taken for a particle to free fall four times the distance. Draw a sketch of the velocity time for the motion. Prove that the initial reaction exerted by the floor on a man's feet when he's standing in the elevator is $\frac{23}{48}$. Find the Reaction on the man during the last descend. (Take g = 10ms⁻²)
- 74. It is wanted to bring a water bucket of mass Mkg upto wall of a well which is at a hm height. Let the maximum tension the rope can withstand safely be 2kMg(k > 1/2). Show that the minimum time taken for the water bucket to reach upto the well wall is $2\sqrt{\frac{kh}{(2k-1)g}}$.

- 75. Two particles P and Q of mass mkg and 2λmkg respectively is attached to two ends of an inextensible string which goes through fixed smooth pulley A. The system is released by keeping the two particles hm above the ground. Find the tension of the string.
 - i. Find the time taken and the velocity with which the particles hit the ground.
 - i. If the horizontal plane is inextensible, find the time taken to hit the ground and come to rest and start the motion again.
 - iii. If $\lambda = 1$ and h = 2m, deduce that the time the particle Q is at rest on the ground is $\frac{2}{15}\sqrt{30}$. Let g =10ms⁻².





78. A particle Q of mass 2mkg is on an inextensible horizontal ground. The inextensible string attached to it is attached to a particle P of mass 3mkg and goes around a fixed smooth pulley A and the system is released from rest. The height from P to the inextensible floor is am. Show that the common acceleration of the system is $\frac{g}{5}$ ms⁻² and also obtain the tension of the string.

Show that the maximum height the particle Q reach is $\frac{6a}{5}$ m and the time taken



79.

An inextensible string attached to a point O in the ceiling goes around a moveable smooth pulley A of mass mkg and around a fixed smooth pulley B hanging from the ceiling and the other end is attached to a particle P of mass Mkg. The free parts of the string are vertical. The system is released from rest at the moment the P particle is at a height of hm from the ground. Show that

the time taken for the particle P to come to the horizontal ground is $\sqrt{\frac{(4M+m)h}{(2M-m)g}s}$.

When M = 3 and m=1, deduce the time taken by the P particle to come to the ground is $\sqrt{\frac{13h}{5g}}$ s.







horizontal plane. The string goes around a smooth pulley P and attached to a particle B of mass 3mkg. The coefficient of friction between the plane and A is 1/3. The system is released from rest when the particle B is at a height of hm from the ground where the string is stretched.



- i. Tension of the string.
- ii. If the B doesn't bounce back after hitting the floor, and when A is in motion and has not reached P, show that the velocity with which B

reaches the floor is $\sqrt{\frac{14\text{gh}}{15}}$.

iii. Show that the minimum length of the string is $\frac{12h}{5}$.

86. AB is a smooth inclined plane. The particle P on AB is attached to one end of an inextensible string and goes around a smooth pulley A and below the smooth pulley C and above the fixed pulley D and attached to particle Q on the smooth horizontal plane from the other end. Find the acceleration of the particle and the tension of the string. If AB



= am, when the particle P is released close to A, find the time taken to come to B and the velocity at which it reaches B.



α 🌢 λmkg

2mkp

C, show that the displacement of B in 1s is $\frac{5}{7}$ m.

89. Two smooth pulleys A and B of mass M_1 and M_2 respectively are fixed to the ceiling using two slacked vertical rods. As illustrated in the diagram an inextensible string goes around the pulleys A, B and the moveable pulley C of mass m_3 and two particles of mass m_1 and m_2 are attached to either ends. The aparts of the string which are not in contact with the pulleys are vertical. By showing that the tension of the string is $\frac{4m_1m_2m_3g}{[4m_1m_2+m_3(m_1+m_2)]}$ Find the force exerted on the ceiling by the system.



- 90. The inextensible string going through the smooth fixed pulley A is attached to a particle R of mass 3mg and on the other end to a pulley B of mass m. The inextensible string going through the smooth fixed pulley B is attached to two particles P and Q of mass m and 2km respectively. The system is released from rest when the particle Q is $\frac{1}{2}$ m above the ground.
 - i. Write equations to find the acceleration of P, Q, R and B and the tension of the strings.
 - ii. If k = 2, find the acceleration and the tension of the strings. When the Q particle reaches ground, assuming that particles P and R didn't reach the pulley, show that the velocity with which the

particle reaches the ground $\frac{1}{12} \sqrt{\frac{5}{3}} \text{ms}^{-1}$ and the time taken is $\frac{1}{2} \sqrt{\frac{3}{5}} \text{s}$.

91. A light smooth pulley is hung from a light string. The inextensible string which goes around the pulley is attached to a particle of mass 4m from one end and from the other end a second smooth pulley of mass m is attached. The second inextensible string which goes around the second pulley is attached to a particle of mass m from one end and from the other end a particle of mass 2m. As illustrated in the diagram the system is kept at rest and released. Show that the heavier object falls down with an acceleration of $\frac{B}{23}$ then find the acceleration of the other particles. Find the pull on the ceiling.

92. The diagram illustrates an arrangement of an object A of mass m on a smooth horizontal table attached to a particle E of mass m' using a slacked inextensible string ABCDE and small pulleys. The string goes around the smooth fixed pulley B and D. C is a moving smooth pulley of mass M which is held by two parts of the string. The AB part of the string is horizontal and the BC, CD and DE parts of the string are vertical.

If at time t the length of AB and DE aparts are x and y respectively, write the equation for the motion of the masses m, m' and M. Deduce that the tension T is obtained by $T = 3\left[\frac{4}{M} + \frac{1}{m} + \frac{1}{m'}\right]g$. Hence, if $\frac{2}{M} = \frac{1}{m} + \frac{1}{m'}$ show that C pulley is stationary.



95. Two particles A and B of mass 3m kg and mkg respectively are attached to the inextensible string going around the fixed smooth pulley. The particle 3m is above a smooth inextensible table. The system is released from rest. Find the time taken for the particle A to hit the table. Find the time taken for A to begin the motion again. Let $g = 10 \text{ ms}^{-2}$.



99. AB is a plane which is inclined 30° to the horizontal with a coefficient of $\frac{1}{\sqrt{3}}$. A smooth pulley is fixed at B. Let AB = lm. Particles P and Q of mass m and 3m is attached to either end of an inextensible string going around the pulley B, and particle P is at A and the particle Q is at B, and the system is released from rest where the string is slightly stretched. C is vertically below B on the inextensible AC plane.

Pulleys & Wedges



100. A particle of mass m₂kg is on the rough horizontal plane. The inextensible string attached to it goes around the smooth pulley A and is attached to a vertically hanging particle of mass m₃kg. The inextensible string attached to the particle m₂ goes around the smooth pulley B and is attached to a vertically hanging particle m₁. Let the



coefficient of friction be μ . When the system is released from rest,

- i. If $m_1 > m_3 + \mu m_2$, show that the particle m_1 move downwards.
- ii. If $m_3 > m_1 + \mu m_2$, show that the particle m_3 move downwards.
- iii. If $m_1 = 3m$, $m_2 = 2m$, $m_3 = m$ and $\mu = \frac{1}{2}$, what happens to the system? Show that the particle m_1 starts the motion from B and within time $2\sqrt{\frac{l}{g}}$ reaches the ground with a velocity of $\sqrt{\frac{gl}{3}}$.
- 101. The inextensible string attached to the point O in the ceiling goes around the smooth moveable pulley A of mass Mkg and goes above a smooth light fixed pulley B and the other end is attached to scale plate C of mass M. Unit objects D and E each of mass mkg is kept on a scale plate as such on top of each other. The system is released from rest. Find the acceleration of A and C, and the tension of the string. Show that the reaction between D and E is $\frac{(3M-4m)2mg}{(5M+4m)}$. Show that the reaction on the scale plate is $\frac{2m(3M-4m)g}{5M+4m}$. Find the resultant force on the ceiling.
 - One end of an inextensible string is attached to the ceiling. The string goes below a moveable smooth pulley C of mass km and above a fixed smooth pulley and other end is attached to a scale. There are two objects A and B as such that



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116. A wedge with an inclination of α is kept on a horizontal table. A particle is kept at the bottom of that side of the wedge. The wedge is made to move with a constant acceleration F along the table. If F > gtan α , prove that the particle goes up along the inclined side of the wedge. The wedge is in motion in this manner for a period of T, after that it moves with a constant velocity. If T =

 $\left[\frac{2 \operatorname{gh sec} \alpha}{\operatorname{F}(\operatorname{F} \cos \alpha - \operatorname{g} \sin \alpha)}\right]^{\frac{1}{2}}$, show that the particle reaches a vertical height of h along the plane.



ii. Show that $2\dot{x}^2 + 2\dot{y}^2 - 4\dot{x}\dot{y}\cos\alpha - \frac{4gy\sin\alpha}{4gy\sin\alpha}$ is a constant. Also show that the acceleration of the wedge is $\frac{g\sin 2\alpha}{2-\cos 2\alpha}$.



122. A wedge of mass M and an α (< $\pi/2$) inclination is kept on rough horizontal table. Here, let the coefficient of friction be μ . A smooth particle of mass m (\geq M) is projected with a velocity V upwards along the maximum slope line through the face of the wedge. If the wedge moves, show that its acceleration is $\left[\frac{m \cos \alpha \sin(\alpha - \lambda) - M \sin \lambda}{m \sin \alpha \sin(\alpha - \lambda) + M \cos \lambda}\right]$ g. Here $\mu = tan\lambda$ and $0 \leq \lambda < \alpha$. Find the time taken by the particle to return to the projection point.





Pulleys & Wedges





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$B \xrightarrow{\langle \psi \rangle} [\theta 0]$





A light inextensible string passes over a light smooth pulley fixed to the ceiling of a lift and carries particles of masses m and Km (K > 1) at its ends. The lift is made to move vertically upwards with constant acceleration F, and at the same





diagram, with OA being horizontal.

If F is the magnitude of the acceleration of the wedge relative to the floor and f is the magnitude of the acceleration of the particle P relative to the wedge,





142.





AC are lines of greatest slope of the relevant faces and $ABC = ACB = \alpha$. Two smooth particles P and Q of masses m and λ m ($\lambda > 1$) respectively, are

attached to the ends of a light inextensible string. The string passes over the pulley and the particles P and Q are placed on AB and AC respectively, with the string taut as shown in the figure. The system is released from rest. Obtain the equations of motion for the particles P and Q along BA and AC respectively, and for the system horizontally. Show that the magnitude of the acceleration of each of the particles P and Q relative to the wedge is $\frac{(\lambda-1)(\lambda+3)g\sin\alpha}{(\lambda+1)[(\lambda+3)-(\lambda+1)\cos^2\alpha]}$.



The wedge is placed with the face containing BC on a fixed smooth plane of inclination α to the horizontal, with AB horizontal as shown in the figure. Two particles P and Q of masses m₁ and m₂ respectively, placed on AB and AC respectively are connected by a light inextensible string which passes over a small smooth pulley at the vertex A. The system is released from rest with the string taut.



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146. Two particles A and B of masses m and 2m respectively are attached to the two ends, of a light inextensible string of length 2*I*, which passes over a fixed small light smooth pulley C. The system is held with each particle at a depth *I* below C and released from rest in this position. Using the principle of conservation of energy, show that the speed V of each particle after moving a distance x (< *I*) is given by $v^2 = \frac{2gx}{3}$, Hence or otherwise, find the acceleration of the system.



Now, suppose that $\alpha = \frac{\pi}{4}$ and $M = \frac{5m}{2}$ show that the speed of the wedge at the instant when the particle wedge leaves the wedge is $\sqrt{\frac{2ag}{21}}$ 2014 A/L



wedge moves in the direction of \overrightarrow{BC} and that the magnitude of the frictional force exerted on the wedge by the floor is $\frac{R}{6}$,

where R is the magnitude of the normal reaction exerted on the wedge by the floor. Obtain equations which are sufficient to determine R, in terms of m and g.







152. A particle P of mass m in and a particle Q of mass λ m are attached to the two ends of a light inextensible string which passes over a smooth fixed pulley. The system is released from rest, with the string taut, as shown in the figure. The particle P moves downwards with acceleration $\frac{g}{2}$. Show that $\lambda = \frac{1}{3}$. If the particle P strikes a horizontal inelastic floor with speed v and the particle Q never reaches the pulley, find the time taken by the particle Q to reach the maximum height from the instant when the particle P struck the floor.

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A particle P of mass m is placed at the point A and it is given a velocity u along \overrightarrow{AB} , where $u^2 = \frac{7ga}{3}$ as shown in the figure. Show that the retardation of P relative to the block is $\frac{2g}{3}$ and find the velocity of the particle P relative to the block when the particle P reaches B. Also, there is a small hole on the upper face of the block at the point E on BC such that $BE = \frac{\sqrt{3}a}{2}$ By considering the motion relative to the particle P will fall into the hole at E.

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Past Papers 11111 С φ m⊾ а \mathbf{b}^{B} \mathbf{b}^{A} m \overline{m} L $\overset{Q}{D}_{2m}$ P m**Ç** Y Х 3m $\frac{\pi}{3}$ В С M N

