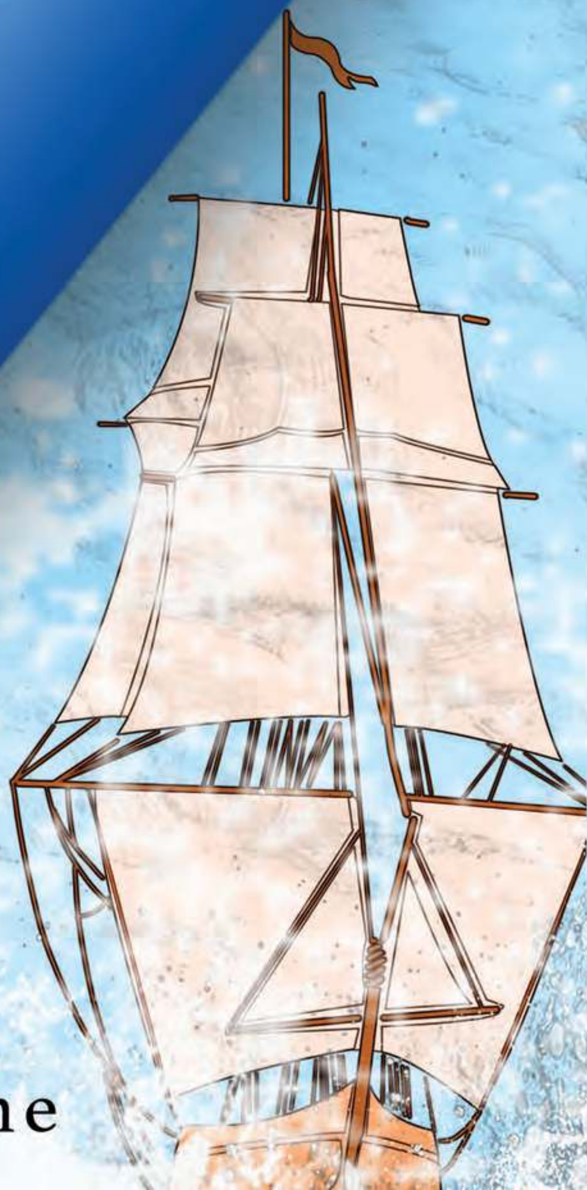
A/L Combined Maths

RELATIVE VELOCITY



Raj Wijesinghe

Relative Velocity

1. Two motor vehicles A, B move at velocities $50ms^{-1}$ and $20ms^{-1}$ respectively towards north.
   i. Find the velocity of A relative to B.
   ii. Find the velocity of B relative to A.

[redacted]

5. Two motor vehicles A, B move at velocities $100ms^{-1}$, $50\sqrt{3}\ ms^{-1}$ towards north and 30° to the west from south respectively.
   i. Find the velocity of B relative to A.
   ii. Find the velocity of A relative to B.

[redacted]

Relative Velocity

[redacted]

[redacted]

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[redacted]

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Relative Velocity

[redacted]

[redacted]

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[redacted]

[redacted]

20. Unit vectors towards east and north are $\underline{i}$ and $\underline{j}$ respectively. A motor vehicle A moves towards west with a constant velocity 20 kmh^{-1}. A cyclist B moves [redacted] $6t\underline{i} + (1 - 8t)\,\underline{j}$]. If the distance between A and B is d km, deduce that $25d^2 = 36\,(100t^2 - 16t + 1)$. Show that the minimum distance between A and B is 720 km and find the time to reach it.

[redacted]

23. A balloon moves vertically upwards with a uniform velocity of u ms^{-1}. Wind [redacted] direction of the balloon. Find the value of θ. If the observer in the balloon sees that the direction of the wind is perpendicular to the direction of the balloon, find the actual direction of the wind.

[redacted] A ship moves towards west at a velocity of 30 kmh^{-1} and the second ship moves towards south at a velocity of 20 kmh^{-1}. A seafarer in ship one sees a ship three moving towards south east and a seafarer in the ship two sees a ship third moving towards the direction 60^0 from north to the west. Show

[redacted]

27. [redacted] aeroplane is $u(\underline{i} + \sqrt{3}\underline{j})$. Find the velocity of the helicopter and find its magnitude and direction.

If the velocity of the aeroplane is $3u\underline{i}$, show that the magnitude of velocity of the helicopter relative to the aeroplane is $2u\ kmh^{-1}$.

[redacted]

[redacted] A ship starts from a point A which is 10km away in the south to the lighthouse L. The velocity of the ship in still water is 10 kmh^{-1}. The ship [redacted]

[redacted]

[redacted]

[redacted]

33. A child moves to the north along a straight road at a constant velocity. The child feels the wind blowing from the direction 60^0 from west to the south. The child turns right and moves along a crossroad with the initial velocity. He feels the wind blowing to the direction 60^0 from south to the west. If the constant velocity of the child is 10 kmh^{-1}, show that the wind blows to south west. Show that the magnitude of velocity of the wind is $5\sqrt{2}(\sqrt{3}+1)kmh^{-1}$.

[redacted]

[redacted] A ship A moves to west at a velocity of 20 kmh^{-1}. A person on the ship feels the wind blowing from the direction $22\frac{1}{2}^{\circ}$ from west to the south. When the ship moves to south with the same velocity, the person feels the wind [redacted]

36. A boat A moves to east at a velocity of 2u ms^{-1}. A boat B is moving at a velocity of u ms^{-1} and the bearing is 030^0. At a certain moment an observer in A sees the boat C moving to south. An observer in B sees the bearing of C as 150^0. Show that the velocity of C is $u\sqrt{7}ms^{-1}$ and find its direction.

37. An observer in the ship A which is moving to north at a velocity of 20 knots, [redacted] boat moving to a direction 30^0 from north to the east. If the enemy boat moves to a direction α from north to the east, show that tanα $= \sqrt{3} - 1$.

38.

i. A person moving towards east feels the wind blowing from the direction α from north to the west. When the person moves to north with the same speed, he feels the wind blowing from the direction β from west to the north. Show that the actual direction of the wind is to the direction θ from north to the west taken from the equation tanθ (1- tanβ) = 1 + tanα.

ii. When the person moves to north, the person feels the wind blowing to the direction α from east to the south. Show that the actual direction of the wind is same as that.

[redacted] Two roads south to north and west to east meet at the junction O. The velocities of the two motor vehicles A, B travelling at the two roads are v_1

kmh^{-1} and v_2 kmh^{-1} respectively. Motor vehicle A moves towards north. The [redacted]

40. A helicopter moves to north at a constant velocity of u kmh^{-1}. The helicopter driver sees a train moving on the ground to south east. The driver found that the railway line is to north east by searching the map. Find the speed of the train. When the velocity of the helicopter decreases to $\frac{u}{2}$ kmh^{-1}, show that the direction of the train relative to the driver is to the east.

[redacted]

43. A child rides a bicycle to north at a velocity of v kmh^{-1}. He feels a wind [redacted] west. Find the actual direction of the wind. Show that the actual velocity of the wind is $v\sqrt{7}\,kmh^{-1}$.

[redacted]

[redacted]

46. A motor vehicle moves to south at a velocity of u ms^{-1}. Driver feels the wind [redacted] north at a velocity of 2u ms^{-1}, the driver feels the wind blowing to the direction ϕ from west to the north. Show that 2tanϕ = 3tanθ – tanα.

[redacted]

49. An aeroplane A moves at a velocity $30(\underline{i} + \underline{j} + 2\underline{k})$ kmh^{-1}. The pilot sees a bird moving at a velocity $-20\ (\underline{i} + \underline{j} + \underline{k})$ kmh^{-1}. Find the velocity of the bird and show that its magnitude is $30\sqrt{2}$ kmh^{-1}.

[redacted]

[redacted]

51. Two roads west to east, south to north intersect at the junction O. Two motor vehicles A, B move in the first and second roads at velocities 30 ms^{-1} and $30\sqrt{3}$ ms^{-1} respectively towards O. At 12 noon, A is situated at the point P which is $100\sqrt{3}$ m away in the west from the point O. B is situated at the point Q which is 100m away in the south from the point O. Mark the path of B relative to A and find the shortest distance between A and B and the time taken to reach that.

[redacted]

[redacted]

[redacted]

55. The position vectors of P, Q relative to O are $8\underline{i} + \underline{j}$, $2\underline{i} + 5\underline{j}$. The unit of displacement is meters. At time t = 0, two objects A, B start at the points P,

Q with velocities $\sqrt{2}u\ ms^{-1}$, 2u $ms^{-1}$and move in the directions $\underline{i} + 3\underline{j}$ and $\underline{i} + 2\underline{j}$.

i. Find the path of B relative to A.

ii. Find the shortest distance and time taken for the shortest distance when u=$\sqrt{5}$.

[redacted]

57. Two straight lines AOB and COD intersect at the point O making an angle 60^0 [redacted] kmh^{-2}. Find the velocity, acceleration and path of Q relative to P. Show that the shortest distance between the two vehicles is 15km. Find the time taken to reach the shortest distance.

[redacted]

[redacted]

[redacted]

60. Two railways meet at a junction inclined α with each other. Two trains move towards the junction on separate railways at velocities u, v. The distances between the junction and the trains at the beginning are a, b respectively. If the two trains come close to each other, (av > bu). Then show that the shortest distance between the two trains is (av – bu) sinα / $\sqrt{u^2 + v^2 - 2uv\cos\alpha}$. Hence show that the value of v for the two trains to collide is $\frac{bu}{a}$. $(v\cos\alpha < u)$

61. Two horizontal and vertical lines intersect at the point O. Two particles A, B move towards O at velocities $10ms^{-1}$ and $10\sqrt{3}\ ms^{-1}$ respectively. At t = 0, A and B particles are initially at $100\sqrt{3}$ m and 100m distances respectively away from O. Find the,

[redacted]

63. Two roads west-east and south-north intersect at the junction O. At t = 0, two motor vehicles A, B are at the points P, Q which are in the directions west and south respectively. A, B move towards O at velocities $u\sqrt{3}\ kmh^{-1}$ and u kmh^{-1} respectively. Here OP = a km, OQ = b km. (b>a)

i. Find the magnitude and direction of the velocity of B relative to A.

ii. Mark the path of B relative to A.

iii. Show that the shortest distance between A, B is $\frac{b\sqrt{3}-a}{2}$.

64. Two straight lines are perpendicular to each other and intersect at the point O. Two particles P, Q are a m distance from the point O. P, Q move towards O at velocities $v\sqrt{3}ms^{-1}$ and v ms^{-1} respectively.

    i. Find the magnitude and direction of velocity of the particle Q relative to P.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

70. The velocities of the two ships A, B are $8(2\sqrt{3}\,\underline{i} + \underline{j})$ kmh^{-1}, $2(\sqrt{3}\,\underline{i} - 3\underline{j})$ kmh^{-1} respectively. The position vectors of them at 12 noon are $4(-\underline{i} + 2\underline{j})$ km and $12(\underline{i} + \sqrt{2}\,\underline{j})$ kmh^{-1}.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted] Two ships A, B are moving at speeds $18\frac{1}{2}$ kmh^{-1} and $20\frac{1}{2}$ kmh^{-1}. Ship A moves towards south, and ship B moves in an unknown straight line. An [redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted] The distance from the edge of the pavement to the straight line l is a m. A motorcycle C moves along the line l at a speed of u ms^{-1}. At t = 0, a pedestrian [redacted]

[redacted]

[redacted]

82. At t = 0, the boat A is 130 m away in the north relative to boat B. The boat B moves to east at a velocity of 13 ms^{-1}. The velocity of the boat A is 5 ms^{-1}. Find the direction that the boat A should be turned in order to get close to the boat B as much as possible and find the shortest distance and the time taken to reach the shortest distance.

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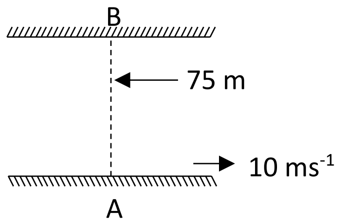

96. A river of width 50 m and having parallel banks flows at a velocity 4 ms^{-1}. A and B are two points on the two banks. The line AB is inclined 60^0 with the upstream. A child who can swim at a velocity of $4\sqrt{3}$ ms^{-1} in still water, starts to move from the point A and swims to the direction θ from the the upstream. The child moves from A to B. Find the value of θ and show that the time taken to move from A to B is $\frac{25\sqrt{3}}{3}\,s$.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

101. A river having parallel banks flows at a velocity of v kmh^{-1}. A child can move at a velocity of $V\sqrt{3}$ kmh^{-1} in still water. A child wants to move downstream from a point P on a bank to a point Q which is on the other bank. The line PQ makes an angle 60^0 with the bank. In which direction should the child swim in order to move from P to Q. If the width of the river is d km, show that the time taken to move from P to Q is $\frac{d\sqrt{3}}{3u}$ h.

[redacted]

103. An aeroplane leaves the airport O to to move to an airport A which is a km away. The velocity of the aeroplane in still air is v kmh^{-1}. A wind blows to a direction 60^0 inclined to the line $\overrightarrow{OA}$ at a velocity of u kmh^{-1}. Show that the direction that the aeroplane needs to be turned in order to move from O to A is $\sin^{-1}\left(\frac{\sqrt{3}\,u}{2v}\right)$ inclined to $\overrightarrow{OA}$. Find the time taken to reach it.

[redacted]

[redacted]

105. A river of width d m flows at a uniform velocity of u ms^{-1}. A person who can swim at a velocity of v ms^{-1} relative to the river, swims perpendicular to the bank. Find the time T in seconds that the person takes to move across the river. Show that the time taken for the person to move d m distance along the bank to the upstream and swim back to the initial point is $\frac{2vT}{\sqrt{v^2-u^2}}$ seconds. Find the reason for v to be higher than u?

106. A river flows from north to the south at a velocity of 5 ms^{-1}. The unit vectors towards east and north directions are $\underline{i}$, $\underline{j}$ respectively. A child starts from the point A in the western bank in order to move towards the opposite point B in the eastern bank at a velocity $\frac{10\sqrt{3}}{3} ms^{-1}$ to the direction α inclined to the north. Show the above velocities in the form $x\underline{i} + y\underline{j}$, hence find the actual velocity of the child. Show that $\alpha = 30^0$. If $AB = \frac{100\sqrt{3}}{3}$, show that the time taken to move from A to B is 20 seconds.

107. A, B are points on the same bank of a river which is flowing uniformly. A boat takes t_1 hours to move from A to the point B. The time taken by the boat to move from B to A is $t_2 (t_2 > t_1)$. Show that the time taken by a wooden block which moves freely on water to move from A to B is $\frac{2t_1t_2}{t_2 - t_1}$.

[redacted]

109. A swimmer takes t seconds to move a m distance towards upstream of a river. The time taken to move the same distance downstream is $\sqrt{3}$t. Show that the velocity of water is $\frac{\sqrt{3}(\sqrt{3}-1)a}{6t}$ ms^{-1}. If the width of the river is a m, show that the time taken to across the river from the shortest path is $(3)^{1/4}$ t seconds.

110. The velocity of an aeroplane in still air is v kmh^{-1}. A wind blows to the direction α from east to the north at a velocity of w kmh^{-1}. Here AB = a km. B is in the east relative to A. Show that the time taken to move from A to B is $\frac{a}{v^2-w^2}\left(\sqrt{v^2-w^2\sin^2\alpha}-w\cos\alpha\right)$ hours. Assuming the time taken by the aeroplane to turn is negligible, Show that the time taken to move in the horizontal square path ABCDA is $\frac{2a}{v^2-w^2}\left[\sqrt{v^2-w^2\sin^2\alpha}+\sqrt{v^2-w^2\cos^2\alpha}\right]$ hours.

111. A, B, C are three airports that are on the vertices of an equilateral triangle. The maximum velocity of an aeroplane in still air is v. When a wind blows to the direction AB at a velocity of u (<v), show that the minimum time that the aeroplane take to move in the path ABCA without stopping is $a\left(\frac{v+\sqrt{4v^2-3u^2}}{v^2-u^2}\right)$. If the aeroplane needs N litres of fuel to move along the path ABCA in still air, find the amount of fuel that is needed in the presence of wind.

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[redacted]

[redacted]

128. A motorcyclist moves at a uniform velocity of u ms^{-1} along a straightline l which is d m away from the edge of the pavement. At t = 0, when the motorcyclist is at the point A, a pedestrian starts to move from the point B infront of the motorcycle on the edge of the pavement at a velocity of v ms^{-1} (v < u). The base of the perpendicular drawn from B to the line l is N where AN = h and BN = d.

i. Mark the path of the pedestrian relative to the motorcycle.

ii. show that if $v > \frac{ud}{\sqrt{h^2+d^2}}$, the pedestrian can move infront of the bicycle without colliding.

129. At t = 0, a football player A moves u ms^{-1} velocity with the ball to the north along a straightline l starting from the point O. at the same moment two players B, C start to run in order to meet A from the point P which is a m away from O in the direction 60^0 inclined with l at the same velocity v ms^{-1}(<u). If they meet A after moving in two different directions, show that the time difference between the times B, C take to meet A is $a\sqrt{4v^2 - 3u^2}/(u^2 - v^2)$.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted] A crow sitting on a branch P of a jack tree flies to a branch Q of a bread fruit tree. The branch Q is 100 m horizontally away from the branch P. The wind blows at a velocity of 10 ms^{-1} perpendicular to PQ at a velocity of $5\sqrt{3}$ ms^{-1}. [redacted]

134. Three points A, B, C on the branches of a mango tree are three vertices of an isosceles triangle. Here AB = AC = a and $\angle BAC = 90^0$. ABC is a horizontal plane. A wind blows to the direction of the median AD at a uniform velocity of U. Two parrots are on the points A, C. The velocity of the parrots X, Y in still air is $u\sqrt{2}$. The parrot X flies from A to B.
    i. Find the time taken t_1.
    ii. If the time taken for the parrot Y move from C to A is t_2, find t_2. Show that $t_2 - t_1 = \frac{a\sqrt{2}}{u}$.

[redacted]

136. A train moves to north at a velocity of u ms^{-1}. A child who is sitting near the window sees a bird which flies a distance of $\sqrt{2}$d m to the north east direction at a velocity v ms^{-1} and returns back to his hand.
    i. Find the time taken by the bird to return back to the hand of the child.
    ii. If the velocity of the bird relative to earth is v, show that the total time taken by the bird to return back to the hand of the child is $2d\,\frac{\sqrt{2v^2-u^2}}{v^2-u^2}$.

[redacted]

[redacted]

139. At 9.00 a.m the ship B is 90 km away in a direction 60^0 from north to the east relative to A. The ship A moves to the direction 30^0 from north to the east at a velocity of $20\sqrt{3}$ kmh^{-1}. The ship B moves to the direction 30^0 from north to the west at a velocity $10\sqrt{3}$ kmh^{-1}. Show that the ships A, B collide and the time that they collide is 12.00pm.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

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[redacted]

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[redacted]

[redacted]

153. An attacking boat moves to north at a velocity of u kmh^{-1}. A submarine moves [redacted]

i. If the shortest distance between the attacking boat and the submarine is S, show that $S = \frac{d(v\cos\theta - u)}{\sqrt{u^2 + v^2 - 2uv\cos\theta}}$.

ii. If the attacking range of the boat is R (R > S), show that the submarine is in danger for $2\sqrt{\frac{(R^2 - S^2)}{(u^2 + v^2 - 2uv\cos\theta)}}$ hours.

[redacted]

155. The velocity of an aeroplane in still air is v kmh^{-1}. A steady wind blows from north at a velocity of kv kmh^{-1} (k < 1). The maximum distance that the aeroplane can travel in still air is d km. Show that the attacking range of the aeroplane to the direction α from north to the east is $d(1 - k^2) / 2\sqrt{(1 - k2\ \sin2\ \alpha)}$ km.

[redacted]

[redacted]

[redacted]

159. The velocity and the displacement of a A warship and a B ship carrying containers are $\underline{V}$, $\underline{r}$ respectively. The velocities and displacements at 12.00 p.m are given below.
[redacted]

160. A ship travels along a straightline l at a uniform velocity of v kmh^{-1}. At t = 0, the ship is at the point A. There is a boat which has get lost is at a point B where its position is on a line 30^0 inclined to l. The maximum velocity of the boat is u kmh^{-1} (u < v). The red flag of the boat can be seen upto a range of r [redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

167. A jet has a steady speed of v kmh^{-1} in still air and an active range (departing and returning) of R_0 km. When a wind of velocity w (< v) kmh^{-1} blows to the north, the active range of the jet moving to the direction θ° from north to the east is R km. Assuming the jet carries fuel sufficient for T hours, show that $R = \frac{R_0}{v}\frac{(v^2 - w^2)}{\sqrt{v^2 - w^2\sin^2\theta}}$. Deduce that $R \leq R_0\left(1 - \frac{w^2}{v^2}\right)^{\frac{1}{2}}$.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

175. A ship sails with uniform velocity having components u and v eastward and northward respectively, relative to water. When the ship is at a distance d north from a submarine, a torpedo is fired from the submarine, with the intention of destroying the ship. Assuming that the torpedo moves uniformly with velocity w relative to water, show that, if the torpedo strikes the ship, then $w > u$, and find the time taken by the torpedo to move from the submarine to the ship.

2005 A/L

176. Water flows in a river, with constant velocity U ms^{-1}, between two straight parallel banks which are d metres a part. A boat moving with speed 2U ms^{-1} relative to water, is required to take a straight course from a point A on one bank to a point B on the other bank and back to A. $\overrightarrow{AB}$ makes a certain acute angle α with the upstream direction of the river and the time from A to B is twice that from B to A.

Draw velocity triangles for the journey from A to B and the return journey, and show that

i. $\sin\alpha = \sqrt{\frac{5}{8}}$,

ii. The velocity of the boat in its journey from A to B, relative to the banks, is of magnitude $U\sqrt{\frac{3}{2}}$.

Deduce the total time taken by the boat to complete the two journeys.

2006 A/L

[redacted]

[redacted]

[redacted]

[redacted] A helicopter, whose speed relative to wind is v km h^{-1} flies around a square track ABCD of side *a km,* in the sense indicated by the order of the letters. A wind blows with velocity w (<v) km h^{-1} in a direction making an acute angle θ with the side *AB.* Assuming that no time is lost in turning round the corners of the track and drawing velocity triangles, show that the sum of the time taken from A to B and the of the track and drawing velocity triangles, Show that the sum of the time taken from A to B and the time [redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

181. The top-most points A, B and C of three lamp-posts lie in a horizontal plane at the vertices of an equilateral triangle of side a. A wind blows in the direction of $\overrightarrow{AC}$ at a steady speed u. A bird, whose speed relative to the wind is v (> u), flies from A to B along AB and then from B to C along BC. Draw the velocity triangles of relative velocities for both parts of the journey **in the same figure.**
Hence, show that the total time taken for the journey from A to C through B is $\frac{4a}{u+\sqrt{4v^2-3u^2}}$

2011 A/L

[redacted]

2012 A/L

183. A van of width b is moving with uniform velocity u along a straight road parallel to the pavement almost touching it. A boy steps onto the road from the pavement at a distance d in front of the van and walks with uniform velocity $v\ (<u \sec\alpha)$ in the direction which makes an acute angle α with the direction of motion of the van. If the boy just escapes without being hit by the van,
Show that $bu = (b\cos\alpha + d\sin\alpha)v$.

2013 A/L

184. A river with parallel straight banks flows with uniform velocity **u**. Two points A and B on either bank are situated such that $\overrightarrow{AB}$ makes an acute [redacted] velocity of the same magnitude $2u$ relative to water to return to A. Sketch

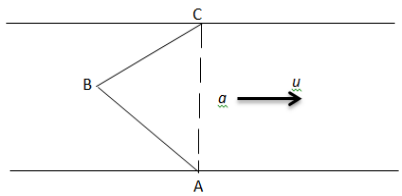
the velocity triangles for the motion from A to B and for the motion from B to A, in the same diagram.
Hence, show that for the motion from A to B and for the motion from B to A, his velocity relative to water must make the same angle θ with $\overrightarrow{AB}$ and $\overrightarrow{BA}$ respectively, where $\sin\theta = \frac{1}{2}\sin\alpha$. If the time taken to swim from B to A is $k(1 < k < 3)$ times the time taken to swim from A to B, show that
$\cos\theta = \frac{1}{2}\left(\frac{k+1}{k-1}\right)\cos\alpha$
Using the above expressions for $\sin\theta$ *and* $\cos\theta$. Show also that
$\cos\alpha = \frac{(k-1)}{2}\sqrt{\frac{3}{k}}$

2014 A/L

[redacted] A ship S sails due North with uniform speed u. Its straight line path is at a perpendicular distance P Eastward from a port P. At a certain instant when the direction of $\overrightarrow{PS}$ makes an angle 45° South of East, two supply boats B_1 and B_2, each moving with uniform speed $v\left(\frac{u}{\sqrt{2}} < v < u\right)$ start from port P at the same instant in two different directions so as to intercept the ship S. These boats reach the ship S at times T_1 and T_2 ($<T_1$), respectively. Given [redacted]

[redacted]

186. A straight river of breadth a flows with uniform speed u. The points A and C [redacted]



adjoining figure.) A boat moving with speed v(>u) relative to water starts off from A and moves until it reaches B. Then it moves from B to C. Sketch the velocity. triangle for the motions of the boat from A to B and from B to C.

[redacted]

[redacted]

[redacted] A ship S is sailing due East with uniform speed u km h^{-1}, relative to earth. At the instant when the ship is at a distance l km at an angle θ South of West from a boat B, the boat travels in a straight line path, intending to intercept the ship, with uniform speed v km h $^{-1}$ relative to earth, where u [redacted]

Show that the angle between the two possible directions of motion of the boat relative to earth is $\pi - 2a$, where $\alpha = sin^{-1}\left(\frac{u \sin\theta}{v}\right)$.

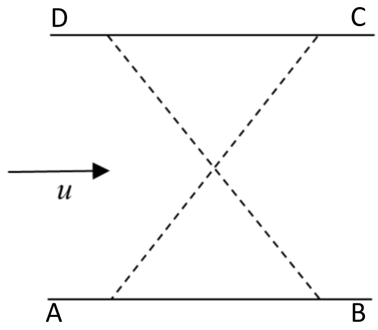
Let t_1 hours and t_2 hours be the times taken by the boat to intercept the ship along these two paths. Show that $t_1 + t_2 = \frac{2lu\cos\theta}{u^2 - v^2}$.

2017 A/L

[redacted] A ship is sailing due North with uniform speed u km h^{-1}, relative to earth. At a certain instant a boat B_1 is observed at an angle β East of South, from the ship at a distance p km from the path of the ship. At the same instant, a boat B_2 is observed Westward at a distance q km from the ship. Both boats sail [redacted]

[redacted]

[redacted] A river of breadth a with parallel straight banks flows with uniform velocity u. In the figure, the points A,B,C and D lying on the banks are the vertices of a square. Two boats B_1 and B_2 moving with constant speed $v\ (> u)$ relative to water begin their journeys at the same instant from A. The boat B_1 first travels to [redacted]



Hence, show that the speed of the boat B_1 in its motion from A to C is $\frac{1}{\sqrt{2}}\left(\sqrt{2v^2 - u^2} + u\right)$ and find the speed of the boat B_2 in its motion from B to D. Further, show that both boats B_1 and B_2 reach D at the same instant.

2019 A/L

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