MATHSCRIBER

A / L Combined Maths

SIMPLE Harmonic MOTION

Raj Wijesinghe

[redacted]

[redacted]

[redacted]

4. The natural lengths two elastic strings OA and AB are 2m, 1m and their moduli of elasticity are 100N, 50N respectively. The strings are connected at A and the point O is fixed to a ceiling. Object of mass 5 kg is connected to B. Object is hanging freely at equilibrium. Find distance to the object from point O. ($g = 10ms^{-2}$)

5. Two strings OA, AB of natural length 1m have modulus of elasticity of 5w, [redacted].

6. An object P of mass 5kg is at equilibrium on a plane 30° inclined to the horizontal by means of an elastic string OP of modulus of elasticity $10N$. If OP is 2m find the natural length of the string. ($g = 10ms^{-2}$)

[redacted]

8. A string of length 2m and modulus of elasticity 50N is connected to fixed-point O on a horizontal celling. The string is connected to a mass of 2kg free

to move up and down vertically. The system comes to equilibrium at point A. Thereafter the string dragged to a point B which is 1/2m vertically below A. Finally, the object is released from B.

i. Find the elastic potential energy stored in the string when the object is at B.
ii. Find the velocity of the object when it reaches A.

[redacted]

[redacted]

[redacted]

12. Displacement of an object $s(t)$ at a time t is given by the function $s(t) = 3t^2 + t$ find the displacement of the object at $t = 1, t = 3$.

13. The angular displacement θ at the time t of an object is given by the equation $\theta = 4t^3 + 2t$. Find the angular velocity of the object when,

[redacted]

14. The equation of motion of an object which moves in a simple harmonic [redacted] of the object when displacement is $2\ m$.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

21. The equation on motion of an object which moves in a simple harmonic motion is given by $\ddot{x} = -4x$ when $t = 0$, $x = 2\text{m}$, $\dot{x} = 4ms^{-1}$.
    i. Find x and $\dot{x}$ when $t = \frac{\pi}{8}$.
    ii. Find $\dot{x}$ when $x = \sqrt{2}\, m$.

22. Object of mass 2kg moves in a simple harmonic motion, the centre of the simple harmonic motion is O. Its periodic time is π and its amplitude is 2m. [redacted].

{When object is reaching P for the second time it has to go to the amplitude A and reach the point P again times taken to reach A from P and P from are equal because the motions are symmetric}

[redacted]

[redacted]

[redacted]

Also show that the amplitude of this motion is given by $\sqrt{\frac{V_1^2X_2^2-V_2^2X_1^2}{V_1^2-V_2^2}}$.

When the object has displacements of $2m,\ 3m$ the velocities of the object are $4ms^{-1},\ 1ms^{-1}$. Show that the periodic time of this motion is $2\pi\sqrt{\frac{1}{3}}\ s$ and that the amplitude of this motion is $2\sqrt{\frac{7}{3}}\ m$.

[redacted]

27. In a large factory a trolley moves horizontally in a simple harmonic motion with an amplitude a and a periodic time T. A small object is placed on the rough surface of the trolley. If the object is at rest relative to the trolley show that the minimum value of the coefficient of friction is given by $\frac{4\pi^2 a}{gT^2}$.

[redacted]

29. An object which moves in a simple harmonic motion around the centre O with a periodic time 10s and with an amplitude of 2m. Find the time taken by the object to travel between two points 1m away from the centre of oscilation O.

30. [redacted]

[redacted]. Show that the periodic time of this motion is given by $\frac{2\pi a}{u}$ also deduce that if $u = 2\sqrt{ag}$ the periodic time is given by $\pi\sqrt{\frac{a}{g}}$.

31. A spring of natural length $a\ m$ and a modulus of elasticity $4mg\ N$ is fixed to a point A on a smooth horizontal table other end of the spring is connected to a object of mass m which lies on the surface of the table. The object is released from a point B which is $4a\ m$ away from A. In a particular moment of this motion the object lies xm away from the point A on the horizontal table, Obtain the equation of motion. Hence show that this motion is simple harmonic and calculate its amplitude and the centre of the simple harmonic motion.

(i) Find the time taken by the object to reach a point $3am$ away from point A .

(ii) Find the time taken by the object to reach a point am away from point A .

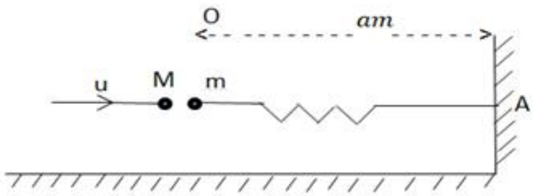
[redacted]

[redacted]

[redacted]

34. The end of a straight horizontal railway is connected to a vertical wall which is perpendicular to the railway. Two-cylinder shaped barriers are connected to the wall just above the railway such that their axes are perpendicular to the wall. The open ends in both barriers are located $d\ m$ away from the vertical wall. The force needed to push a barrier $x\ m$ distance is $\lambda x\ N$ (λ is a constant). A train of mass M moving towards the wall with velocity v hits the two barriers. If V< $d\sqrt{\frac{2\lambda}{M}}$ show that the train doesn't collide with the vertical wall furthermore show that the train is in contact with the barriers for a time of $\pi\sqrt{\frac{M}{2\lambda}}$.

[redacted]



[redacted]

[redacted]

37. An elastic string of natural length 50 m is hung from a fixed-point A on a horizontal ceiling other end of the string is connected to an object of mass 2 kg. The string comes to equilibrium when the length of the string is 70 cm find the modulus of elasticity of the string. Now the object is displaced further 40m downwards from the position of equilibrium. Show that there after object starts to travel in a simple harmonic motion. Furthermore, find the periodic time of the object. (take $g = 10ms^{-2}$)

38. An elastic string of natural length l and a modulus of elasticity mg is hung from a fixed-point A on a horizontal ceiling other end of the string is connected to a object of mass m kg. The object is gently released under gravity from point A. Use conservation of energy in order to find the velocity of the object when the string comes to its natural length. Show that the time where the string is taut is $\sqrt{\frac{l}{g}}[\pi + 2sin^{-1}\frac{\sqrt{3}}{3}]$

[redacted]

[redacted]

[redacted]

[redacted]

42. [redacted]

i. Show that the cylinders won't collide with each other if $u < (l - 2h)\sqrt{\frac{g}{2l}}$.

ii. If $u \geq (l - 2h)\sqrt{\frac{g}{2l}}$ and if the impact between the cylinders are completely elastic show that the periodic time of the buffer is given by $\sqrt{\frac{2l}{g}}\left[\pi - \sqrt{\frac{g}{2l}}\cos^{-1}\left(\frac{l-2h}{u}\right)\right]$.

43. Two elastic strings of natural lengths $2a$ and a with modulus of elasticity $4mg$ and $2mg$ are connected to an object P of mass $m\ kg$ the other ends of [redacted]

motion of the object and deduce that the periodic time of the object is given by $2\sqrt{\frac{a}{2g}}\left[\tan^{-1}\frac{4}{3}+\frac{1}{\sqrt{2}}\left(\pi-\tan^{-1}\frac{4\sqrt{2}}{3}\right)-\tan^{-1}\frac{4}{5}+\tan^{-1}\frac{2\sqrt{2}}{5}\right]$.

44. A force of $m\omega^2(OP)$ directed towards O, acts on a object P of mass "m" which travels on a straight line. ω is a constant. The object is starting motion from rest from the point A, when the object is at a distance x away from the point O the velocity of P is "V", show that $V^2=\omega^2(a^2-x^2)$.

45. A force of $m\omega^2(OP)$ directed towards O, acts on a object P of mass "m" which travels on a straight line. ω is a constant. The object is starting motion from rest from the point A, when the object is at a distance x away from the point O the velocity of P is V, show that $V^2=\omega^2(a^2-x^2)$. On this equation $a=OA$.

[redacted]

46.

[redacted]

b) Object P travels in the OXY plane such that at a time t its displacement OP is given by $\overrightarrow{OP}=a\cos\omega t\underline{i}+b\sin\omega t\,\underline{j}$. (a,b,ω) are constants and i,j are unit vectors along X and Y axes. Show that object P describes an ellipse and find the vertical and horizontal components of the velocity and the acceleration of P. Furthermore, show that projections of describe a simple harmonic motion of periodic time $\frac{2\pi}{\omega}$ on $\overrightarrow{OX}$ and $\overrightarrow{OY}$.

47. [redacted] a simple harmonic motion also obtain the periodic time of this motion. Furthermore, find the thrust of the spring is compressed by a length of $3a$.

48. An elastic string AB of natural length l and a modulus of elasticity $4mg$ is hung from a fixed-point B on a horizontal celling which is located at a distance $2l$ above the ground level other end of the string is connected to an object of mass $m\ kg$. The object is gently released under gravity from point B. By applying conservation law of energy.

49. When a force W is applied on an elastic string of natural length a, it creates an extension of length b on the string. End A of the string is connected to a [redacted].

[redacted]

[redacted]

52. Two particles P and Q of mass m are connected to each other by a light elastic string of natural length l and elastic modulus λ. At the beginning, at $t = 0$, the P particle is at rest. The particle Q is projected from P with a velocity u . By considering their motions relative to the center of masses of P and Q, or otherwise, find the velocities of the particle.

[redacted]

53. [redacted] opposite directions on both particles A and B away from each other. Find the maximum extension of the string and show that the string obtains its maximum extension at a time $\frac{\pi}{4}\sqrt{\frac{3am}{\lambda}}$. When $\lambda = mg$ show that the time for the displacement is $\frac{\pi}{4}\sqrt{\frac{3a}{g}}$.

[redacted] A light elastic string with a natural length a and modulus of elasticity mg is placed on a rough horizontal plane by attaching A of mass M and a B of mass m at both ends, respectively. The coefficient of friction between the table and both masses is μ . Initially, particle B was held at point L at a [redacted]

[redacted]

[redacted]

56. [redacted] O and gently released from rest. Find the kinetic equation of motion and show that the period of this motion is given by $2\pi\sqrt{\frac{l}{6g}}$.

[redacted]

[redacted]

[redacted]

59. A, B, C, D are 4 fixed points on a straight line. A particle P starts to travel along this straight line. The equations of motion of this particle P when the object is OA, AB, BC is given by $\ddot{x} = -\omega^2 x$, $\ddot{x} = 0$, $\ddot{x} = -\omega^2 x$. In the above equations x is the distance measured to the particle P from the point O, ω [redacted]

60. A force of $m\omega^2(OP)$ directed towards O, acts on a object P of mass “m” which travels on a straight line. ω is a constant. The object is starting motion from rest from the point A, when the object is at a distance x away from the point O the velocity of P is “V”, show that $V^2 = \omega^2(a^2 - x^2)$. On this equation $a = OA$.

An elastic string of Natural length $6a$ is connected to two pints A and B which are $9a$ distance apart. An object of mass “m” is connected to string on the trisection point of the closer to A. Now the object is displaced to a point P which is located a distance away from A and then gently released from rest. Show that the object comes to instantaneous rest when the object reaches point $\left(\frac{9+\sqrt{30}}{3}\right)a$ away from A.

[redacted]

62. The height of a cork, which is in the form of a circular cylinder, is hm. The weight is w. The cork floats on a large body of water while its axis is vertical. Find the submerged height. The density of water is ρ. Show that the motion is simple harmonic when pressed down vertically x distance from the equilibrium position. Show that the maximum gravity S in the cork is $2\pi\sqrt{\frac{hS}{g}}$.

63. The modulus of elasticity of an elastic string is mg. The natural length of it is l. One end is connected to the fixed-point A. There are two particles of masses m, M kg are attached at the other end. When the system is in equilibrium, the M particle is gently removed from the string. If $M < m$, show that the motion of the particle m is simple harmonic and the periodicity of this motion is $2\pi\sqrt{\frac{l}{g}}$. Also show that the amplitude for this motion is $\frac{lM}{m}$. Show that when $M = 2m$ and M is removed, the string gets slack and that the m particle rises and hits A.

64. One end of an elastic string of natural length a and modulus mg is attached to a point O on a smooth horizontal table, where O is located at a distance [redacted].

If the length of the elastic string is x, show that the system satisfies the equation $\ddot{x} + \frac{g}{2a}(x - 2a) = 0$. Furthermore assume that this system also satisfies the equation $x - 2a = A\cos\omega t + B\sin\omega t$ where A and B are [redacted].

65. AB, BC are two light elastic strings connected at B to form a single string whose natural length is a but whose modulus of elasticity is different. Its A end is fixed to a point on the horizontal ceiling and at the other end C it carries a particle of mass m. It is known that the period of small vertical oscillations around the equilibrium position of a particle is equal to the period of small oscillations of a simple suspension. The particle is projected vertically down from A at a velocity of $2\sqrt{ag}$. Show that the particle returns to A after a time $2\sqrt{\frac{a}{g}}[2\sqrt{2}-2+\pi+\cos^{-1}\left(\frac{1}{3}\right)]$. (I think the elasticity modules should be given).

66. The natural length of an elastic string is $2m$ and its modulus of elasticity is $10Nm^{-1}$. One end of the elastic string is attached to point A on a smooth [redacted].

67. The natural length of an elast elastic string $1m$ and the modulus of elasticity is 5Nof an elastic string. The string is connected to a fixed-point A on a horizontal celling. A particle of mass $200g$ is connected to the other end of the string. When the string is at natural length, the particle is at B and given a velocity of 5ms^{-1} in the direction of $\overrightarrow{AB}$. Find the maximum extension of the string. Find the time taken by the particle to reach B again.

68. One end of an elastic string with a modulus of elasticity of $10N$ and a natural length of $2m$ is mounted on a smooth horizontal surface and the other end is connected to a particle B of $500g$. The particle is released from [redacted].

69. A string of modulus of elasticity $2mg$ is connected to a smooth horizontal plane at the point A. The natural length of the string is l. At the end B of the string the particle of mass mkg is fixed. At position $AB = 3l$, the $m\,kg$ [redacted].

70. The modulus of elasticity is mg N and the natural length is am of an elastic spring. The end A of the spring is connected to a fixed-point and the other [redacted].

71. A particle moves along a straight line so that its acceleration is in the [redacted].

Such a particle moves at a distance of $14m$ from O its velocity is $96\ cms^{-1}$ and its distance from O is $30cm$ and its velocity is $80cms^{-1}$.

[redacted]

ii. [redacted].

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

77. [redacted] the string at a ums^{-1} velocity away from A. The coefficient of friction is μ.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

85. A horizontal platform moves vertically. x is displacement of the plane measured from a fixed horizontal plane at time t, it is given that $x = a\ sin^2\omega t$. ω and a are positive constants. Show that the motion of the platform is a simple harmonic motion with the center of oscillation at $x =$ [redacted].

[redacted]

[redacted]

88. [redacted] a speed of $2ms^{-1}$ from the position of equilibrium. Show that the particle describes a simple harmonic motion and furthermore evaluate the amplitude and period of the simple harmonic motion.

89. [redacted] The boy gently jumps off the bridge starting from rest. Show that the time taken for the child to reach instantaneous rest is given by $\frac{1}{5}(5\sqrt{2} + 2\sqrt{5}\pi - 2\sqrt{5}\cos^{-1}\frac{\sqrt{14}}{7})$. Also calculate the tension of the string at that moment. $g = 10ms^{-2}$.

[redacted]

[redacted]

92. [redacted]. when $\rightarrow \propto$, what is the limit of this time interval?

93. [redacted]. Describe what happens if the particle is vertically pulled down a distance $2a$ from its equilibrium position and then released from rest and $l > \frac{3a}{2}$.

94. [redacted]. Show that the modulus of elasticity of the string is $4mg$ and that the particle once again returns to the level of point A after a time $\sqrt{\frac{l}{g}}\left[2\sqrt{2} - \pi - cos^{-1}(\frac{1}{3})\right]$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

99.

a) A spiral spring is used to hang a mass from a fixed-point. When the mass is at rest, the extension of the string is l. Find the total number of oscillations per second when the mass is given a vertical motion. Modulus off elasticity of the spring is λ .

[redacted]

[redacted]

101. [redacted], when the system is at equilibrium the particle compresses the spring by length of $\frac{a}{4}$. A second particle with the same mass of m is held at rest at a height of $\frac{3a}{8}$ and is released gently from rest such that it collides with the particle connected [redacted]

[redacted].

102. [redacted] If it is given that $l = \sqrt{(v^2 + 3ag)\frac{a}{g}}$, therefore, write a statement for the time it takes for the particle to ascend from this point, hit the ceiling and then return to the lowest point. Assume that the coefficient of restitution between the celling and the particle is e.

103. One end of a light elastic string is of natural length lm, is attached to point A on the ceiling. The other end of the string is connected to particle, after the particle is connected and when the system is at equilibrium the length of the string is am. The particle is displaced vertically downwards until the string length of the elastic string is $4lm$ and then the particle is released gently from rest. The particle just reaches A. Show that $a = \frac{17l}{8}$. Show that the total time taken by the partical from the moment it was released and to reach the point A is given by $\sqrt{\frac{l}{g}}[\sqrt{2} + \frac{\sqrt{3}}{2}(\pi - \cos^{-1}\frac{3}{5})]$.

[redacted]

[redacted]

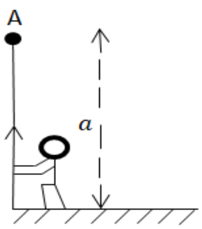
[redacted]

i. [redacted] of a to the level of the point B when the string is extended.

ii. If the projection velocity of the particle at O is $2\sqrt{2ag}$, then assume that the velocity of the particle is v when the length of the string is $a + x$, therefore obtain the equation $av^2 = 2ag(x + 4a) - gx^2$.

iii. Find the maximum distance to the P particle below O. Also calculate the maximum tension of the string.

[redacted]



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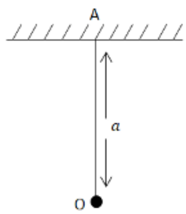

[redacted]

108. A and C are two fixed points. $AC = 3a$ and AC is vertical. A is above the horizontal level of C. AB, BC are two elastic strands with modulus of mg each. A, C are connected to two fixed-points and the particle of mass m is attached to point B connecting both strings to the particle. Particle B is released from rest at the point C. Show that the particle reaches its maximum height from the point C in $\frac{\pi}{2}(1+\frac{1}{\sqrt{2}})\sqrt{\frac{a}{g}}$ time.

109. One end A of an elastic string with a natural length a and a modulus of elasticity of $20mg$ is connected to a fixed-point. The particle m is attached to the end B of the string. The particle is gently released from a point $\frac{a}{2}$ [redacted].

[redacted]

[redacted]

[redacted]

[redacted]

112. [redacted] Show that the time taken by the particle to reach the point O is given by $[\pi - \cos^{-1}(\frac{1}{3}) + 2\sqrt{2}]\sqrt{\frac{a}{2g}}$.

[redacted]

114. AB and BC are two light, elastic strings of different moduli of elasticity and of the same natural length a, that are joined at B to form a single string. The end of A is fixed to a point on the horizontal ceiling and the other end carries a mass of m particle. Oscillations around the equilibrium position of the particle has the same periodic time of a simple suspension of length l. If m is projected vertically down wards from A at a speed of $2a\sqrt{\frac{g}{l}}$, show that the particle comes back to point A at a time $\frac{4a}{\sqrt{gl}}\{\sqrt{1+\frac{l}{a}} - 1 + 2(\pi - \alpha\}\sqrt{\frac{l}{g}}$. Where $\alpha = \cos^{-1}(\frac{l}{l+2a})$.

Show that the string gets slack at a time $\sqrt{\frac{l}{2g}}[\pi - \cos^{-1}\frac{2}{3}$ when the particle is released from rest at a point O vertically below A such that $OA = \frac{9l}{4}$.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

121. [redacted]. Show that the particles collide after a time of $\pi\sqrt{\frac{ma}{2\lambda}}+\frac{a}{u}$.

[redacted]

123. The natural length of an elastic string is $2am$. The particle mkg is fixed at the midpoint O. Both ends of the string A and B are attached on two smooth points with A and B on a smooth horizontal plane. Such that $AB = 2bm$. When $100N$ is applied to the string, it extends 10 cm per one meter. The particle is displaced by a distance of d away from the point of O towards the point A and it is gently released from rest from at that point. If it is given that $(d < b - a)$ Show that the motion of the particle is simple harmonic, and the periodic time of the particle is given by $\frac{\pi}{10}\sqrt{\frac{am}{5}}$.

[redacted]

[redacted]

[redacted]

[redacted]

129. [redacted]. If it is given that $OP = x$ at time t, after the object is projected, Write down the equations of motion for the particle,

[redacted]

[redacted].

130. There is a particle P of mass m on a smooth plane. The particle P is attached ends of two strings. The other ends of the strings are connected to fixed- [redacted].

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

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[redacted]

[redacted]

[redacted]

141. [redacted].

[redacted].

i. In the case when $a < \frac{1}{2}$, find the period and the amplitude of the ensuing motion.

[redacted]

[redacted].]

2007 A/L

[redacted]

Find the maximum height reached by the particle P above the initial position and show that the time taken to reach this height is $\sqrt{\frac{a}{g}}\{\pi - \alpha + 2\sqrt{2}\}$, where α is the acute angle $\cos^{-1}\left(\frac{1}{3}\right)$.

2008 A/L

143. [redacted].

[redacted].

2009 A/L

144. [redacted].

[redacted].

[redacted].

Show that, the particle P performs simple harmonic motion for a time $\sqrt{\frac{a}{\dot{g}}}\left(\frac{\pi}{2}+\ \alpha\right)$, where $\alpha = \sin^{-1}\left(\frac{a}{b}\right)$ and the velocity of the particle P at [redacted].

2010 A/L

[redacted]

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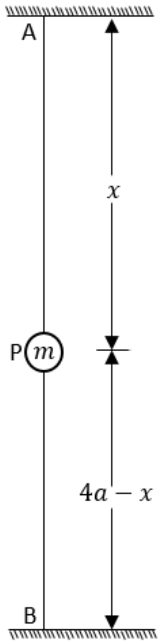
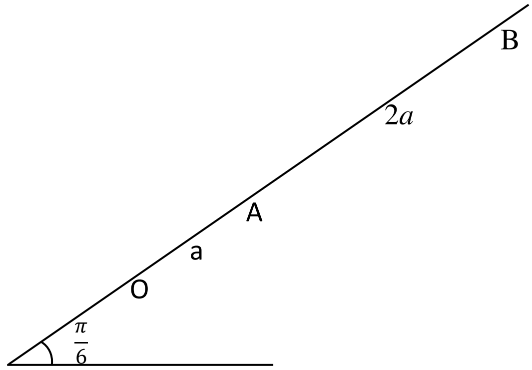
152. [redacted].

[redacted]



Re-write this equation in the form $\ddot{X} + \omega^2 X = 0$, where $X = x - \frac{5a}{2}$ and $\omega^2 = \frac{2g}{a}$. Using the formula $\dot{X}^2 = \omega^2(c^2 - X^2)$, find the amplitude c of this motion. At the instant when the [redacted]

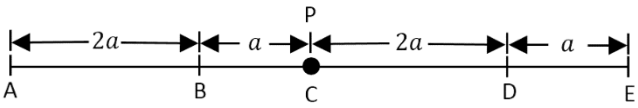
153. [redacted] and AB = 2a. One end of a light elastic string of natural length a and modulus of elasticity mg is attached to the point O and the [redacted]. [redacted], where $\omega = \sqrt{\frac{g}{a}}$. Find the center of the above simple harmonic motion and using the formula $\dot{y}^2 = \omega^2(c^2 - y^2)$,find the amplitude c and the velocity of P when it reaches A. Show that the velocity of P when it reaches O is $\sqrt{7ga}$. Show also that the time taken by P to move from B to O is $\sqrt{\frac{a}{g}}\left\{cos^{-1}\left(\frac{1}{5}\right) + 2k\right\}$, where $k = \sqrt{7} - \sqrt{6}$.



[redacted].

2019 A/L

154. [redacted]



[redacted].

[redacted].

[redacted].

[redacted].

[redacted].

[redacted]

2020 A/L