A/L COMBINED MATHEMATICS

STATICS II

EQUILIBRIUM

Maths Simplified !

RAJ WIJESINGHE

[redacted]

2. [redacted] through A keeping AB horizontal. Find the tension of the string and the reaction at A.

[redacted]

4. The plane surface of a hemisphere of radius 3am is fixed on a horizontal plane with its curved surface upwards. A sphere of radius 2am and [redacted] tension of the string. Given that OAC is the force triangle.

[redacted]

6. [redacted]
the upward horizontal and kept on a smooth peg which is at C where AC= [redacted] points.

[redacted] Edge of a smooth bowl of radius am is kept horizontal. End A of a uniform rod AB with length 2a and weight W is kept inside the bowl and the point [redacted]

[redacted]

[redacted]

9. [redacted]

curved edge of the lamina touches the wall. OD = 3a. Find the tension in the string and the reaction force at the contacting point. D is the point that lamina touches the wall.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

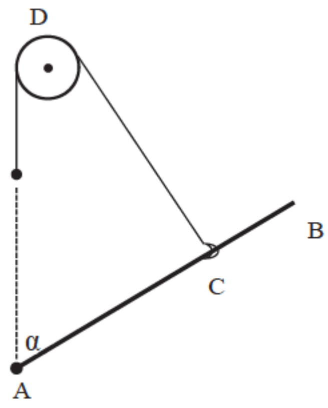
[redacted]

[redacted]

16. End A of a uniform rod AB of length 2m and 3W weight is hinged to a [redacted] the string is attached to a weight 5W. Find the value of α and the reaction at A, when the system is in equilibrium.

[redacted]

[redacted]



i) Show that the string CD is perpendicular to AB.

ii) If the rod AB is inclined α to the vertical, show that $\tan\alpha = \frac{3w}{2W}$.

[redacted] Find the condition for the equilibrium and the minimum value [redacted]

[redacted]

[redacted]

[redacted]

[redacted] the cone. The string goes over a peg and the cone hangs in equilibrium. If the axis of the cone is horizontal and the cone is at rest, show that the length [redacted]

[redacted]

[redacted]

[redacted] A rod AB of weight W is kept fully inside a smooth hemispherical hollow bowl of centre C and radius r. Bowl is fixed with its brim horizontal. The rod is divided into two parts of length a and b at the centre of gravity G. Here b > a and r > $\sqrt{ab}$. If the inclination to the horizontal of the rod at [redacted]

[redacted]

[redacted]

inclination θ of the ladder with the horizontal is given by $\tan\theta = \frac{k-\mu^2}{\mu(k+l)}$

[redacted]

[redacted]

string at the point B. The string goes over a smooth peg at point A and the other end of the string is attached to the end C of 3W weight rod CD. The

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

i) Draw the force diagrams when $\theta = 0^0$ and $\theta = 90^0$.

[redacted]

[redacted]

[redacted]

[redacted]

ii) Show that the three forces are equivalent to a couple if and only if $\lambda = \mu = \gamma \neq 1$.

39. Forces of magnitude 3P, 4P, 5P act along the order of the letters AB, BC, CA of an equilateral triangle ABC of length of a side a. Find the magnitude, direction of the resultant and the point where the resultant meets the extended BC. A couple of magnitude $2\sqrt{3}Pa$ in the direction of BAC and a force Q are applied on the force system. If the new system is in equilibrium, find the magnitude, direction, and the line of action of force Q.

40. Forces of magnitude (X_r, Y_r) $x_r, y_r)$ r= 1,2,.....n acts at the points (x_r, y_r) in the (x,y) plane. Show that the algebraic sum of the moments M around the point P(x,y) is $M = G - Yx + Xy$. Here $X = \sum_{r=1}^{n} X_r$, $Y = \sum_{r=1}^{n} Y_r$ and $G = \sum_{r=1}^{n}(x_r Y_r - y_r X_r)$. If the system is equivalent to a single force, deduce the equation of the line of action of the resultant. In a coplanar system of forces moments around the points (2, 1), (0, 0), (-3, 4) are 11, 5, -15 units, respectively. If the given system makes a couple with the addition of a force at (3, 2), show that the equation of the line of action of the new force is 2x – y – 4 = 0. Find the moment of the couple.

41. a) Find the resultant of two parallel forces P, Q. If the two forces are parallel and opposite in direction does the proof valid? Show that a couple can be equivalent to another couple, but a single force cannot be equivalent to another force. Do you consider couple has no relationship with a force and it is independent?

b) System of parallel coplanar system of forces of magnitude $F_i = (i = 1, 2, \ldots . n)$ are on the Ox, Oy axes acting at the points (x_i, y_i) respectively. Every force is inclined θ with the Ox axis. Show that the resultant goes through a fixed point for any value of θ.

42. The coordinates of the vertices of a rectangle ABCD are (0, 0), (0, 2), (-2, 3), (-4, -1) respectively. The forces $\overrightarrow{AB}, 3\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$ act along the sides of the rectangle. Find the magnitude and the equation of the line of action of the resultant. Find the point at which the resultant cuts the Ox axis.

43. Show that a system of forces on the plane XOY can be equivalent to a couple of moment G and a single force (X, Y) which acts the point O. If any of X and Y becomes nonzero, show that the system is equivalent to a single force which acts on the line Yx – Xy – G = 0.

44. In a coplanar system of forces anticlockwise moments of magnitude 5, 1, 0 are acting at the points (0, 0), (1, 0), (2, 1) respectively. If the addition of the components along the positive x axis be X and the addition of components along the positive y axis be Y, show that X=3 and Y=4. Hence find the magnitude and the equation of the line of the line of action of the resultant.

45. In coplanar system of forces if addition of the moments of three non collinear points is zero, show that the system is in equilibrium.

The angle bisectors of a triangle ABC meet at the point I. Forces P, P, λP, μP, νP act along the order of the letters BC, CA, BA, IB and IC respectively. If the system is in equilibrium, find the values of λ, μ, ν.

If $\lambda \neq 2$, $\mu = 2\lambda \cos\frac{B}{2}$, $\nu = 2\cos\frac{C}{2}$, show that the system of forces is equivalent to a single force and find its magnitude, direction and the line of action.

46. A force system consists of forces of magnitude (X_i, Y_i) act at the points $A_i \equiv (x_i, y_i)$ in the xOy plane. If the system is equivalent to a couple G_o and a single force (X_o, Y_o) acting at the point O. Here i = 1,2, ...n. If $X_o^2 + Y_0^2 \neq 0$, show that the system is equivalent to a single resultant force R. If the force R goes through (-1, -1) and (4, -2) find X_o ,Y_o and G_o , given that $|R| = \frac{1}{2}$.

47. Three forces of magnitude P, λP, λ^2P act along the sides of a triangle ABC in the order of the letters BC, CA, AB. If the resultant force goes through the orthocenter of the acute angled triangle, show that $\frac{1}{\cos A} + \frac{\lambda}{\cos B} = \frac{\lambda^2}{\cos(A+B)}$. Deduce that λ should be negative value.

48. The center of the triangle ABC Δ is G. Forces $3\overrightarrow{BG}, 3\overrightarrow{CG}, 3\overrightarrow{GA}, 6\overrightarrow{CB}$ act along the directions BG, CG, GA, CB respectively. Show that the resultant is parallel to CG and find the line of action of the resultant.

49. ABC is a triangle. P is point on the plane of the triangle. The center of the triangle is G. Show that the resultant of the forces represented by $\overrightarrow{PA}, \overrightarrow{PB}, \overrightarrow{PC}$ is given by $3\overrightarrow{PG}$.

The in center of the triangle ABC is I. The length of the sides BC, CA, AB are a, b and c respectively. Show that the resultant of the forces represented by $a\overrightarrow{PA}, b\overrightarrow{PB}, c\overrightarrow{PC}$ is $(a+b+c)\overrightarrow{PI}$.

[redacted]

51. A particle of weight w kg is kept on a rough slanted plane. The angle [redacted]

52. A weight of w kg is kept in equilibrium on a rough plane which makes an angle of θ with the horizontal. Show that the minimum force required to move the mass upwards the plane is wg sin $(\theta + \lambda)$. Let the Coefficient of friction angle be λ. Keeping the direction of the force as constant, show that if θ > λ then before the mass move downwards the force can be reduced to wg $\sin(\theta + \lambda)\ \sec 2\lambda$.

[redacted]

[redacted]

[redacted]

[redacted]

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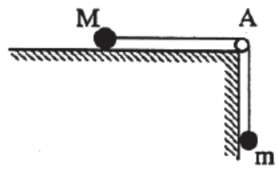
[redacted]

[redacted]



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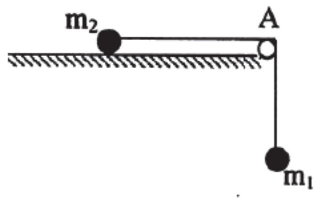

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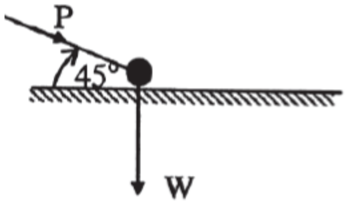
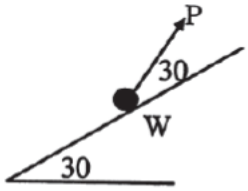


[redacted] A W weight particle is in equilibrium on a plane 30° [redacted]



[redacted]

[redacted]

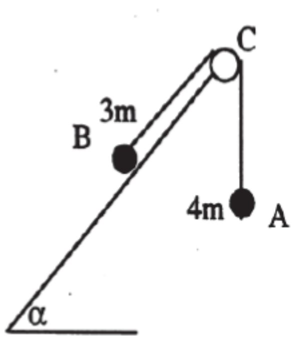
[redacted]

[redacted]

[redacted]

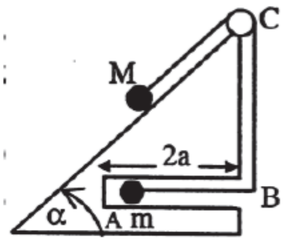
70. The planes AB and BC makes angles of 60° and 30° with the horizontal, respectively. AB is smooth and BC is rough. The coefficient of friction of the BC plane is $\frac{1}{\sqrt{3}}$. Find the velocity with which it hits the inelastic floor. Find the velocity at which particle Q approaches B.

71. The particle B is on a rough plane. The mass is 3m. Let the coefficient of friction be $\frac{1}{2}$. The inextensible string attached to the particle B goes through a smooth pulley C and vertically hangs a mass 4m. When the system is released from rest, find the common acceleration of the particles. Particle A falls a distance of 5am and hits the inelastic floor.



If the particle B doesn't reach the pulley, find the time and the distance for the particle B to come to rest.

72. AB is a rough fixed circular hole. Let BC and CD be smooth fixed planes. CD makes an angle of α with the horizontal. Let be AB = 2a and CD = 4a. Find the velocity at which the particle m reaches B when the system is released from rest. Let μ be the coefficient of friction between the AB plane and the particle m. When the particle m reaches B, the string is carefully severed. Find the velocity at which the particle M arrives at D, if the particle M starts from C.



[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

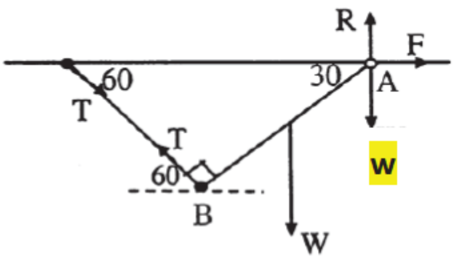
[redacted]

[redacted]

[redacted]

[redacted]

[redacted] A ring of weight w is fixed at the end A of a uniform rod of length 2a and weight W. [redacted]



[redacted]

[redacted]A [redacted] other on a rough vertical plane. Let the coefficient of friction between the floor and the log and wall [redacted] be μ. Find the angle the log [redacted]

[redacted] fixed in a ka distance away from A. $(0 \leq k \leq 2)$. AB rod is in [redacted]

[redacted]

83. A [redacted] to a point P vertically above A. Let $P\widehat{R}A = 90^0$. The rod is in the vertically perpendicular plane. The rod makes an angle of α to the vertical and AP = d. If the rod is about to slip downwards from A, show that $d = a \tan \alpha (\sin \alpha + \mu)$. Coefficient of friction between the wall and rod is μ.

[redacted] makes an angle of θ with horizontal and end A is in contact with the [redacted]

85. [redacted].

86. A [redacted]

angle of θ with the horizontal, show that $\mathrm{p} = \mathrm{W} \tan\theta$. For the side AB to not slip from the peg, show that the minimum value of μ should be $\frac{1}{\sqrt{p}}$.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

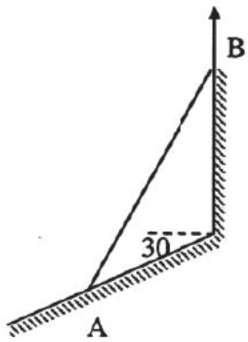
[redacted]

[redacted]

[redacted]

[redacted]

98. [redacted]



reactions of the points of contact A and B are $\frac{2\sqrt{6}W}{3}$ and $\frac{W\sqrt{3}}{3}$ respectively.

[redacted]

100. [redacted] between the rod and the ring A be μ. When $A\hat{B}O = 90^0$, show that $\mu = \frac{\tan\alpha}{2+3\tan^2\alpha}$. Deduce that when $\alpha = 30^0$, $\mu = \frac{\sqrt{3}}{9}$.

[redacted] Length 2a weight 3w regular OA rod is smoothly hinged at O. An end of an inextensible string with W weight is attached to a ring which [redacted]

[redacted]

[redacted]

[redacted]

104. The curved surface of a uniform hollow hemisphere touches a horizontal plane and rough vertical wall, plane of the edge is inclined 45^0 to the horizontal. The hemisphere is in limiting equilibrium. The coefficients of friction between hemisphere and horizontal plane, hemisphere and vertical plane are $\mu_{1,}\mu_2$. Show that $1 - 2\sqrt{2}\mu_1$. If $\mu_1 = \mu_2 = \mu$, deduce that $\mu = \sqrt{2\sqrt{2} - 1} - \sqrt{2}$.

[redacted]

weight, radius and friction angle are W, a and λ respectively. Show

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted] A uniform solid sphere of weight W is rest on a rough vertical [redacted]

[redacted]

[redacted], show that $\theta \leq \sin^{-1}\left(\frac{\tan\lambda}{\tan\alpha}\right)$.[redacted] Here λ ($< \alpha$) is the friction angle. Find [redacted]

[redacted]

[redacted]

If the inclination of the rod with the horizontal is $\theta(<\alpha)$ and a [redacted]

113. Define the terms 'friction angle' and 'friction cone'

a. A uniform rod is at rest inside a fixed hollow hemisphere. The rod is in limiting equilibrium. If the rod makes a right angle with the center of the hemisphere, show that the inclination of the rod with the horizontal is $\tan^{-1}\left(\frac{2\mu}{1+\mu^2}\right)$, where coefficient of friction is μ.

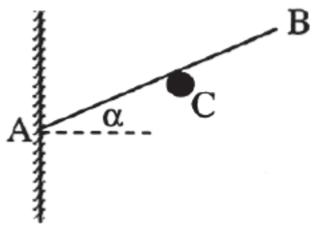
[redacted]

[redacted]

[redacted]

[redacted]

[redacted]



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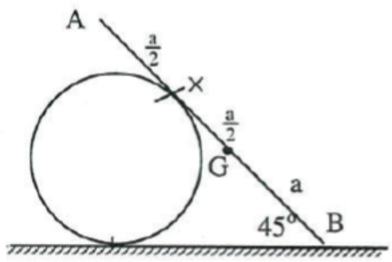

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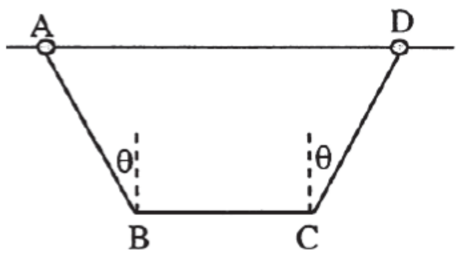

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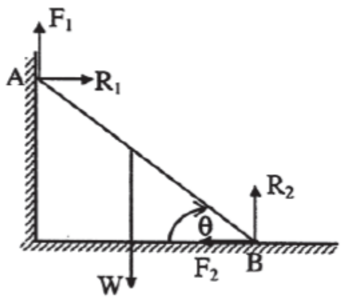

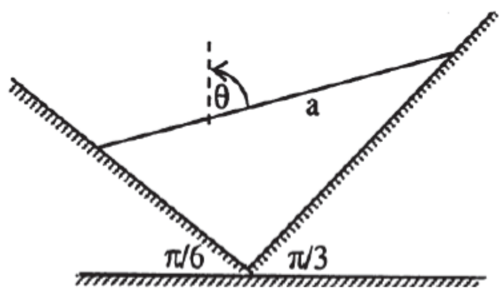

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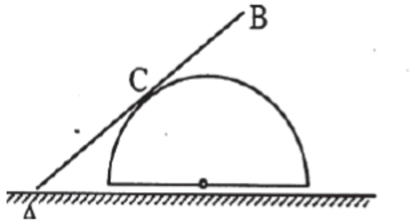

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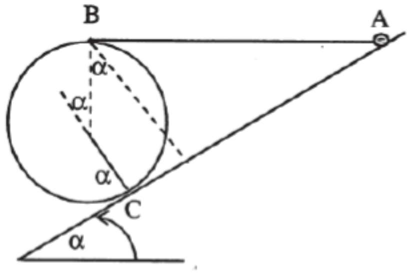


[redacted]

[redacted]

[redacted]

[redacted]

A. Coefficient of friction is μ. The sphere is in limiting equilibrium to [redacted]

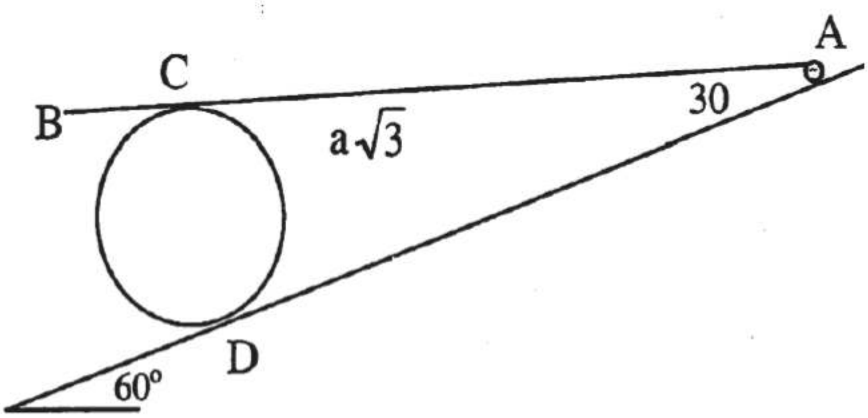
132. [redacted]. If $\frac{l}{3a} = \mu$, show that $T = \frac{W}{3\mu}\sqrt{l + 3\mu^2}$.

[redacted]

[redacted]

[redacted]

[redacted]



[redacted]

[redacted]

[redacted]

[redacted]

140. [redacted]

rod. The coefficient of friction between each ring and the rod is $\frac{1}{2}$. Show that the distance between the rings cannot be less than 4a/5 at equilibrium. Find the maximum and minimum distances that can have between the two rings.

[redacted]

142. [redacted] of the ring is $\frac{W}{3}$ at equilibrium. θ is the angle between rod and vertical wall.

[redacted]

[redacted] horizontal. A child with weight W, climbs the ladder starting from the [redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

150. End A of a uniform rod of length 2a and weight 3W is kept on a rough horizontal plane. The rod is in equilibrium making an angle 45 with horizontal by an inelastic string attached to the other end of the rod. The other end of the string is attached to a ring of weight W which can freely move along a rough horizontal rod. The rod, horizontal rod and the string are in the same vertical plane. The angle between the string and the rod is α where $\alpha < 45$. Show that the point C slips first.

i. If the coefficient of friction at both the contacting points is μ, find the condition for equilibrium.

ii. If $\mu = 1/5$, deduce that $\alpha_{\min} = \tan^{-1} 6/11$

[redacted]

[redacted]

[redacted]

minimum distance between two rings is $\frac{a}{\sqrt{1+4\mu^2}}$. Show that the minimum and maximum distances between the two rings are [redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

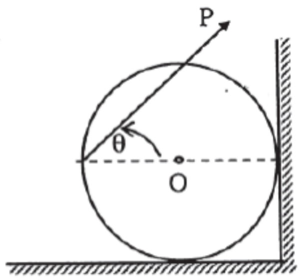
[redacted] The curved surface of a uniform solid hemisphere is kept touching

[redacted]

[redacted]

[redacted]

165. A uniform rod AB of weight W and length 2a is on a rough horizontal plane. An inelastic string attached to B end is going over a fixed smooth pulley D. D is on the same vertical plane of AB. The BD string is inclined α to the horizontal. If the angle of inclination increases gradually, the rod slips on the plane, show that $\mu < \cot\alpha$. If not show that the rod rolls around A. when the rod roll around A and inclined with an angle α to the horizontal, the BD string incline β with the horizontal. If A is about to slip, show that $\mu(\tan\beta - 2\tan\alpha) = 1$.



[redacted]

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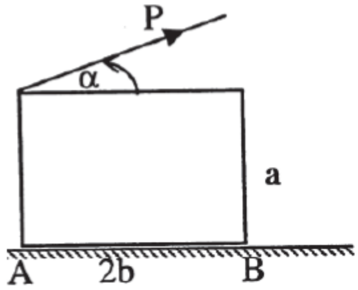

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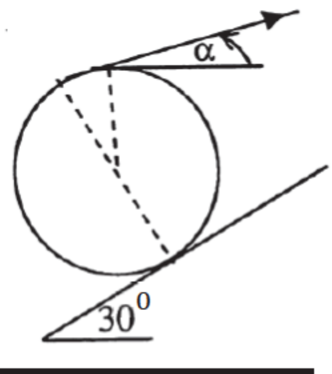

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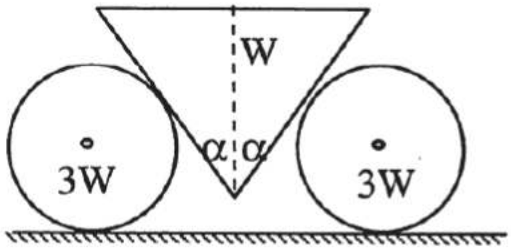

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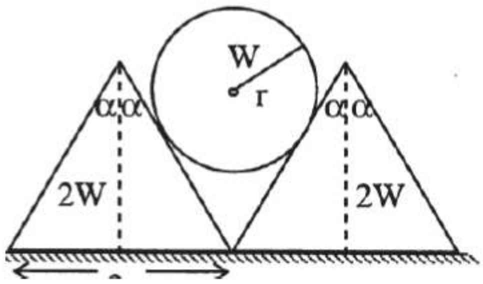

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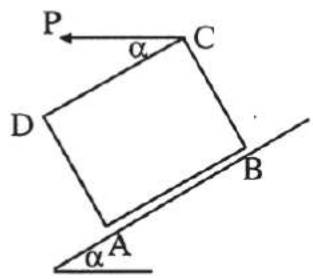

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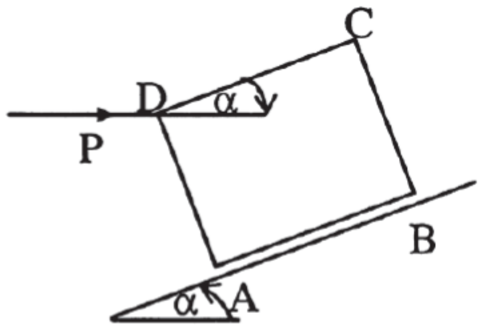

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the upward vertical and it lies in a vertical and it lies in a vertical and

[redacted]

[redacted]

[redacted]

191. [redacted] between R and W. Hence, show that the length CB is $\frac{1}{4}(7l - \sqrt{l^2 + 32a^2})$.

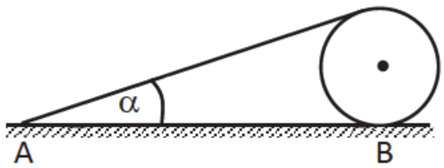
(A/L – 2003)

192. [redacted] both points where friction acts and 2α(b $\leq$r cotα) be the inclination [redacted]

points where friction acts. Show that $(b + x)\sin^2\alpha\cos 2\alpha = r\sin\lambda\cos\lambda$.

(A/L – 2004)

193. [redacted]n between the particle and the floor, show that the tension in the string is $\frac{\mu w}{(\cos\alpha+\mu\sin\alpha)}$. By taking moments about B, find the value of G.



(A/L – 2005)

[redacted]

[redacted]

(A/L – 2006)

195. A uniform rod of length a and weight W rests in a vertical plane inside a fixed rough hemispherical bowl of radius a. The rod is in [redacted]

(A/L – 2007)

[redacted] a) A uniform solid hemisphere of weight W is placed with its curved surface on a rough plane inclined in an angle α, to the horizontal. It is in limiting equilibrium with its plane face horizontal, when a small [redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

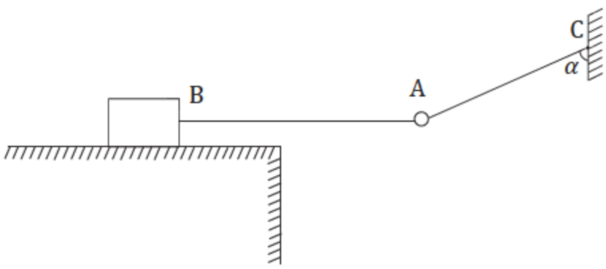
199. A weight W is suspended by two light inextensible strings of lengths a and b from two points at the same horizontal level which are at a [redacted].

(A/L – 2011)

[redacted] A uniform rod AB of weight W and length $2a$ is in equilibrium with the end A on a rough horizontal ground and the end B against a [redacted]

[redacted]

[redacted]



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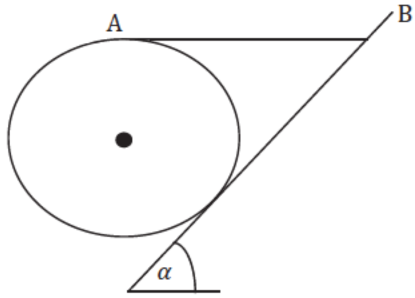


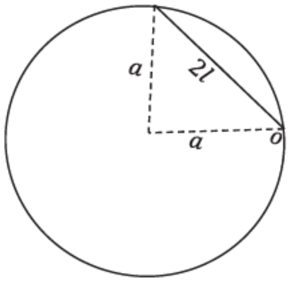
[redacted]

203. [redacted] a vertical place perpendicular to the wall, making an acute angle θ with the wall. Show that for the rod to be in equilibrium in this position, the coefficient of friction μ between the rod and the floor must satisfy $\mu \geq \frac{1}{2}\tan\theta$

(A/L – 2015)

[redacted]

rigid circular wire of radius $a\,(>\sqrt{2}l)$ which is fixed in a vertical plane. A small smooth bead of



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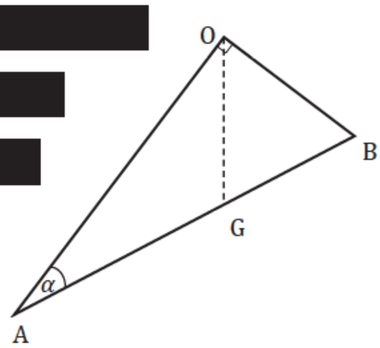

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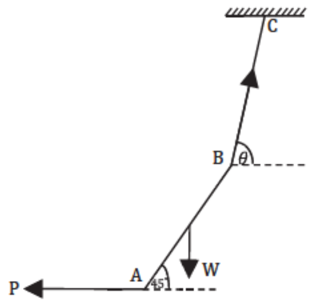

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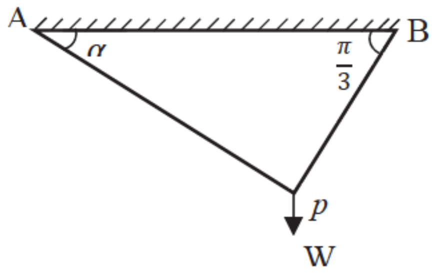
[redacted]

angles α and $\frac{\pi}{3}$ with the horizontal,



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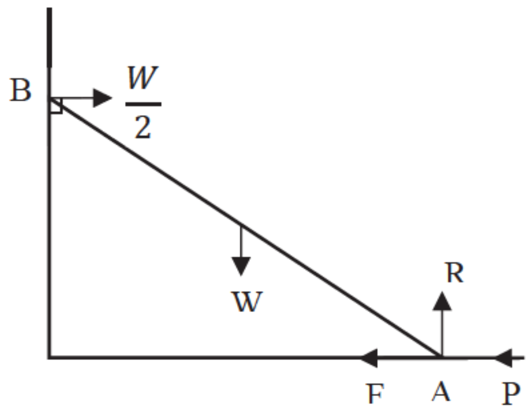

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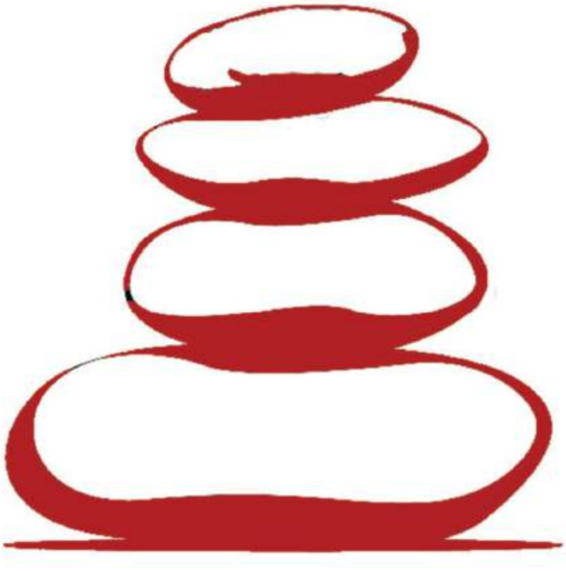

Maths Simplified!

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