A/L Combined Mathematics

STATICS III

Smooth JOINTS & FRAMEWORKS

Raj Wijesinghe

Smooth Joints

[redacted]

[redacted]

3. Two uniform rods of same length and weights $W, 2W$ are joined smoothly at B. AB, BC rods are α inclined to the horizontal by an inelastic string connecting the mid points of AB and BC. The A and C ends are on a smooth horizontal plane while ABC is on a vertical plane. Find the components of the action of B, reaction of A and C and the tension of the string.

   Show that the action of B is $tan^{-1}\left(\frac{1}{6}\tan\alpha\right)$ degrees to the horizontal plane.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

9. [redacted]

[redacted]

[redacted] Four uniform rods of length $2a$ and W weight each are smoothly hinged at the points A, B, C, D. A light rod connecting the midpoints of BC, CD rods and system is hanging from A so that angle [redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

15. AB, BC, CD uniform rods each of length 2a and W weight are freely hinged at the points B and C. AB and CD are supported by a smooth [redacted] of $\tan^{-1}\left(\frac{\operatorname{Tan}\alpha}{3}\right)$ with the horizontal.

16. AB and AC are two similar uniform rods each of weight W hinged freely at A. The B, C ends are attached to a light inelastic string. B and C are at rest, placed symmetrically on two smooth planes inclined at an angle α to the horizontal each. BC is placed horizontally and A is located above BC. ABC is in a vertical plane. Find the reaction of B. Show that the tension of the string is

$W(\tan\theta - 2\tan\alpha)/2$ if $\tan\theta > 2\tan\alpha$ with angle $BAC\measuredangle = 2\theta$. Find the reaction of A.

[redacted]

[redacted]

19. Six uniform rods of length 2a and W weight are smooth hinged from the ends. AB is stationary and horizontal. The system is vertical and shaped as the hexagon ABCDEF by light inelastic unending strings through smooth rings at the points A,

[redacted]

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[redacted] AB uniform rod of length $2l$, weight W is smoothly hinged from the A end to a light rod AC of length $2\ l$. The point B is in contact with a rough vertical wall with coefficient of friction μ. C is pivoted below [redacted]

[redacted] AB, AC are two similar uniform rods of length $2a$ and W weight freely hinged to make a triangle with the uniform rod BC of w weight. Angle $ABC\measuredangle = 2\alpha$. At equilibrium, BC is horizontal, A is [redacted]

[redacted]

[redacted] AB, BC are two uniform rods of length $2a$ and $W, 2W$ weights. They are smooth hinged at B and supported to its equilibrium by rough horizontal pegs P and Q. $PQ = a$, ABC are on a vertical plane with [redacted]

[redacted]

[redacted]

[redacted] A uniform cylinder of 3W weight is at equilibrium lying on a rough surface with its axis horizontal. The rough surface is α inclined to the horizontal. A horizontal uniform rod AB of $2W$ weight is smoothly pivoted by its end A to a point on the inclined surface so [redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted] AB and BC are two uniform rods of length $2a$ and weights $2W$ and $4W$ smooth hinged at B and hung at the point A. The system is at [redacted]

36. AB and BC are two uniform rods of length $2a$ and weights $3W$ and $2W$. The rods are rigidly fixed to each other so that the angle $ABC\measuredangle = 90^0$ is a right angle. Show that the AB makes an angle $tan^{-1}(\frac{2}{7})$ to the vertical and find the tension of the string.

[redacted]

[redacted]

[redacted]

[redacted]

42. Two uniforms rods of equal lengths are smoothly hinged at B. The weights are W and $2W$ each. A and C points are in contact with a [redacted] components of the action at B hinge. Show that $\mu_1 \geq \frac{3}{5} tan\alpha$ and $\mu_2 \geq \frac{3}{7} tan\alpha$.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

47. Three uniform rods of equal lengths and weight are smoothly hinged by the ends forming a triangle. An inelastic string connects [redacted]

48. Two uniform rods of equal lengths a and weight W are smoothly hinged at B. Two smooth pegs P and Q support ABC on a horizontal [redacted]

[redacted] Two uniform rods of length $2a$ and weights $2W, W$ are smoothly hinged at B so that the angle ABC is a right angle. The point A is a fixed, smooth pivot. A string of length $2\sqrt{2}a$ connects the points A [redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted] Four uniform rods of length l and weight W are smoothly hinged at A, B, C and D. Two smooth pegs with a distance between of $2b$ are [redacted]

54. Four uniform rods of length a and W weight are the smooth hinged at the points A, B, C, D and hung by A. An inelastic string connecting AC forms a square of the system. Two inelastic strings of length $2a$ attached to the points B and D hangs a particle with a weight of W. Show that the tension of the string is $\frac{W(35+\sqrt{7})}{14}$. Find the horizontal and vertical components of the action at C hinge.

55. AB, BC, CD are uniform rods of length $2l$ and W weight, smoothly hinged at the points B and C. Two smooth pegs in the same [redacted] the reaction at the B hinge. If the action of B is β inclined to the vertical show that $\tan\alpha \tan\beta = 3$.

56. Four uniform rods of length a and W weight are smoothly hinged at the points A, B, C, D to form a rhombus. The system is hung by two

[redacted]

at the point L show that the distance from L to CD is $\frac{\sqrt{3}a}{4}$. Show that

$CL = a\frac{\sqrt{7}}{4}$.

[redacted] AB, AC uniform rods of length $2a$ and W weight are smoothly hinged at A. A smooth light free moving D ring on the AC rod and a

[redacted]

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[redacted] Three identical light rods are smoothly hinged at A, B, C. Points A, C are horizontal on a smooth support and the plane ABC is [redacted]

[redacted]

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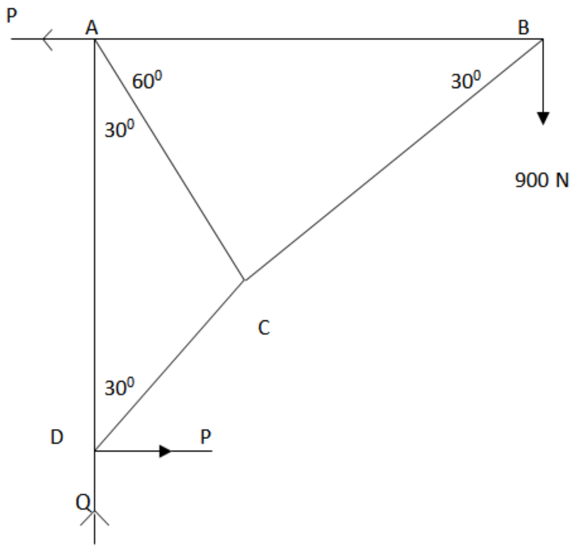

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Frameworks

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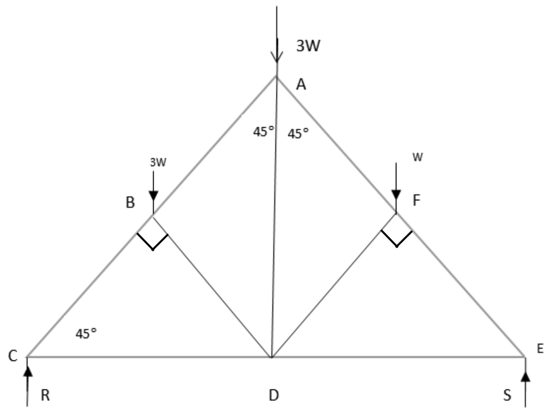

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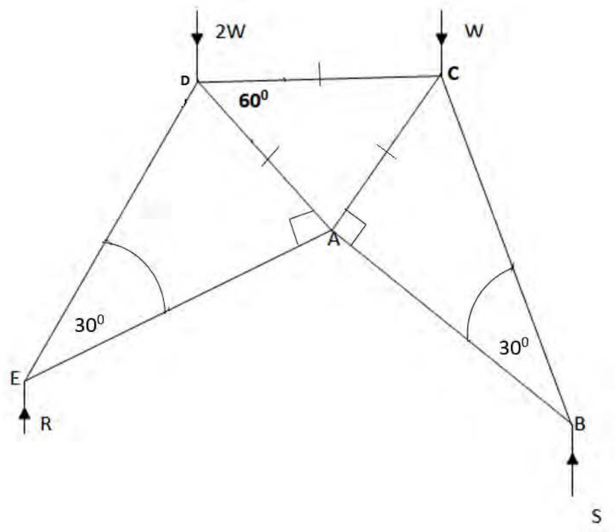

Frameworks

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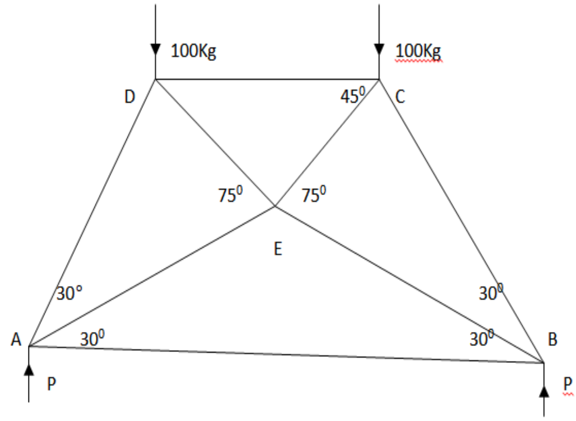

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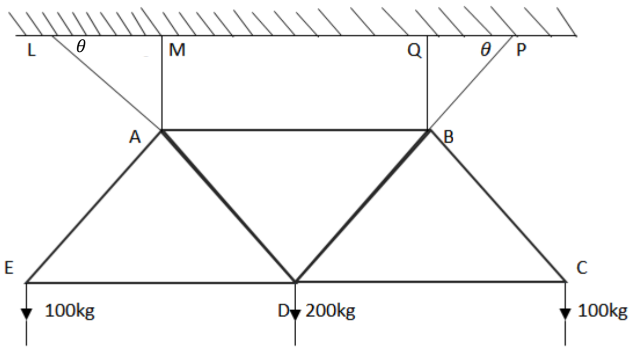

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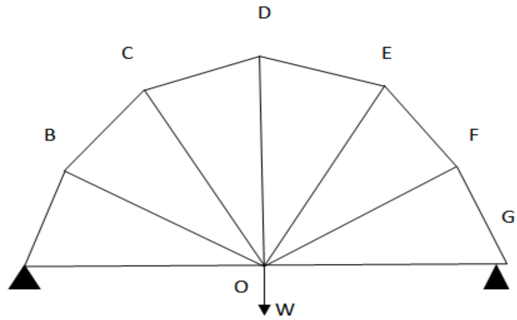
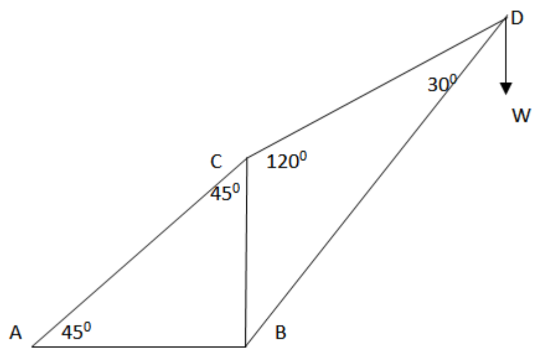
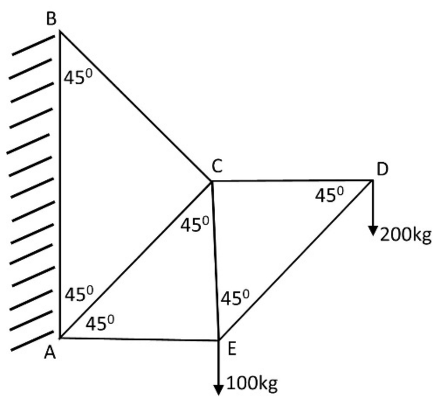


[redacted] The figure shows a vertical framework with five light rods [redacted] at the point D. This framework is [redacted]



[redacted] Rods AC and BC are smoothly hinged. Find the stresses of the rods and find whether [redacted]



Frameworks

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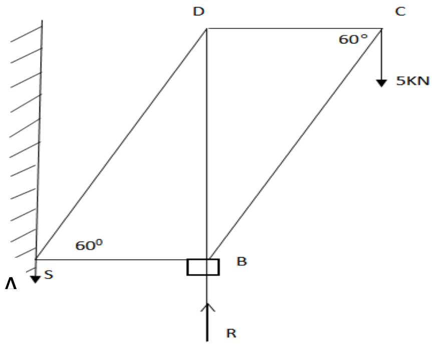

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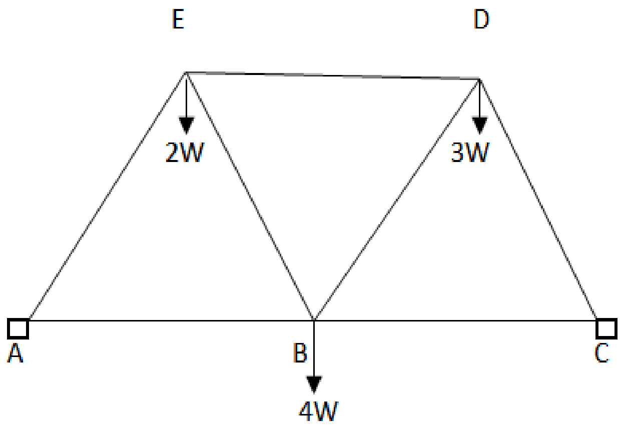

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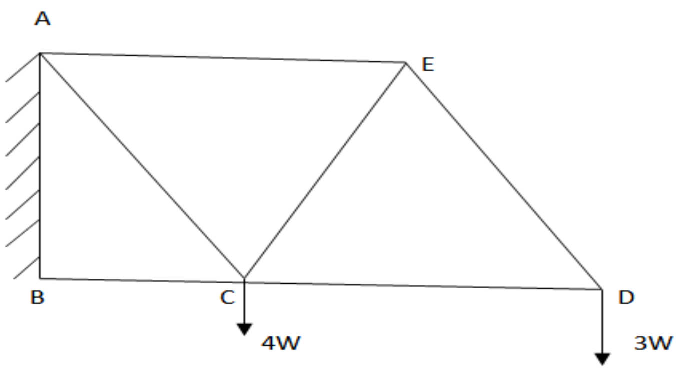
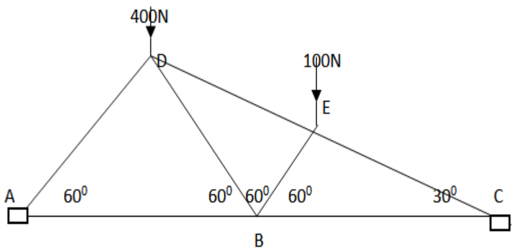
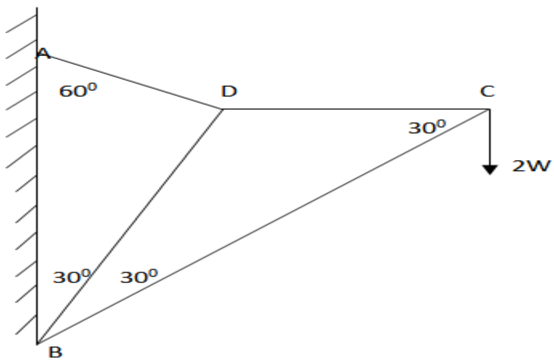


74. [redacted] whether they are tensions or thrusts.



[redacted] The figure shows a framework in the vertical plane made by [redacted]. There is a 2W load at the point C. Indicating the rods that exist under tension, and the rods that exist under thrust. Find the stresses of the rods, whether they are tensions or thrusts and the [redacted]



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Frameworks

[redacted]

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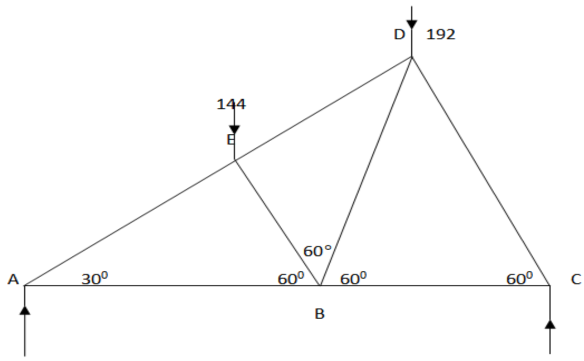

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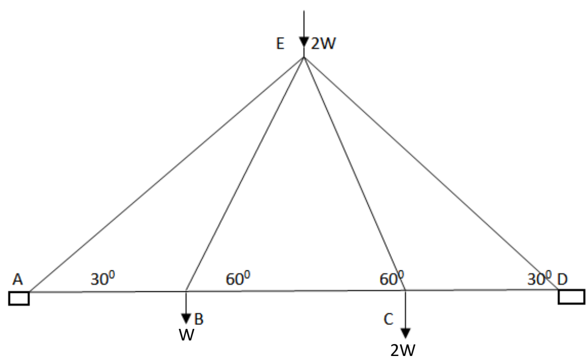

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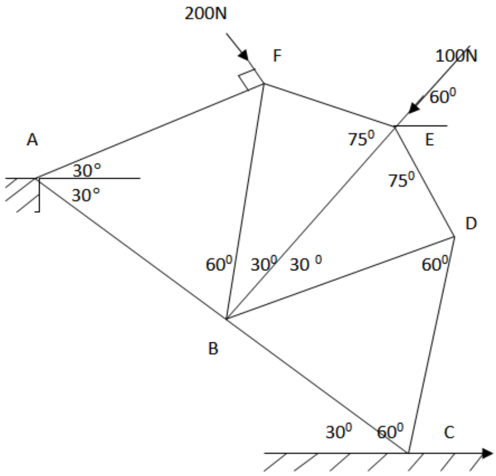

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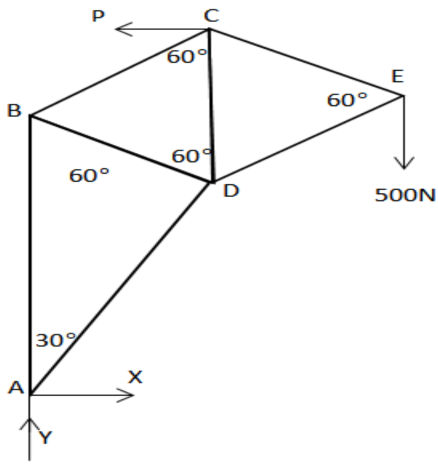

Frameworks

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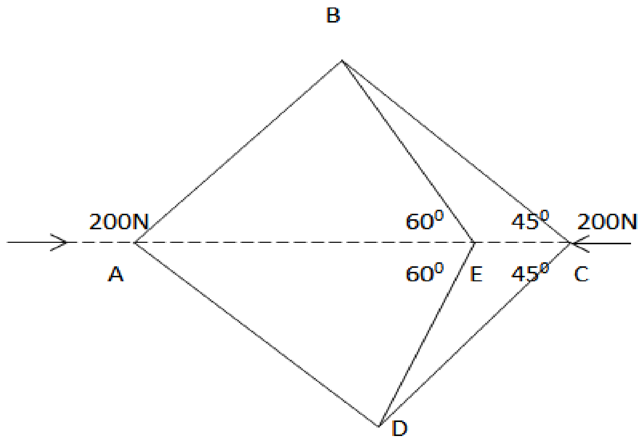

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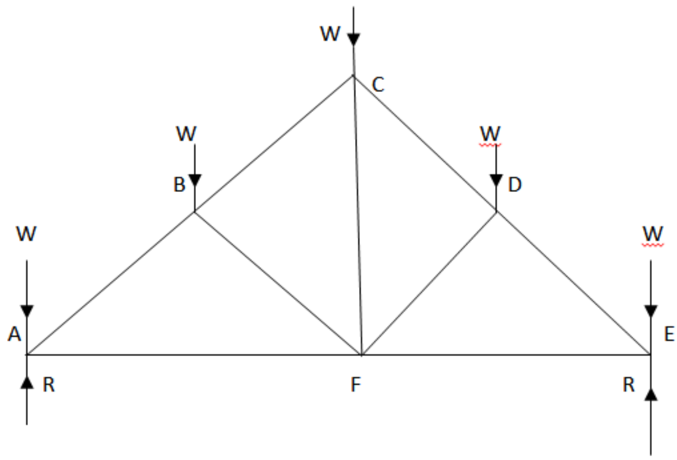

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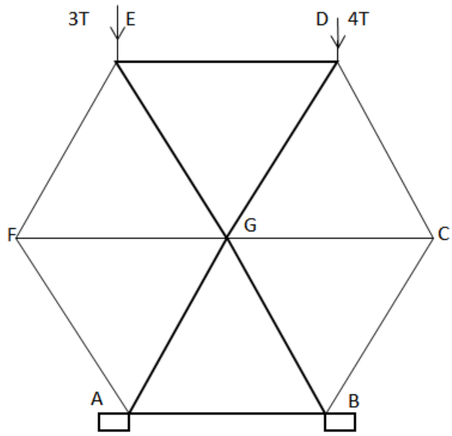

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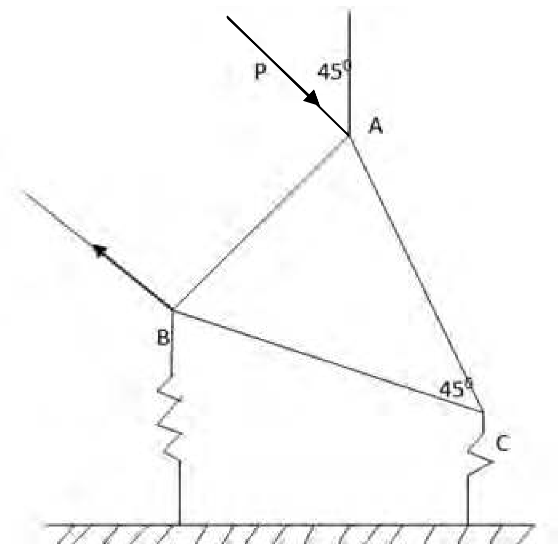

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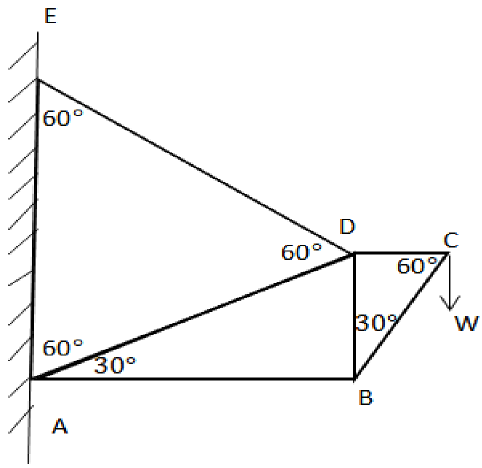
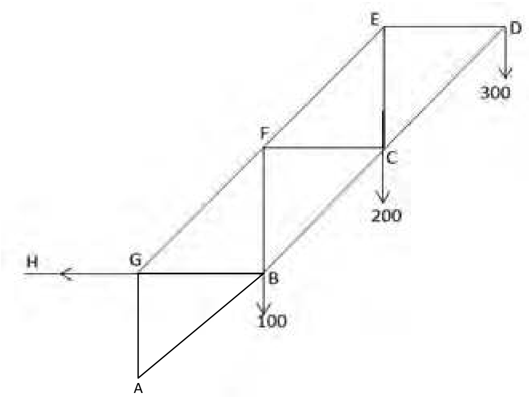


87. A framework ABCDEFG made of smoothly hinged light rods is loaded as shown in the figure. Each of its triangles are right angled isosceles triangles. This framework is fixed so that it can be rotated around A and it is in equilibrium using a chain. Determine the tension of the chain and stresses graphically or using calculations.



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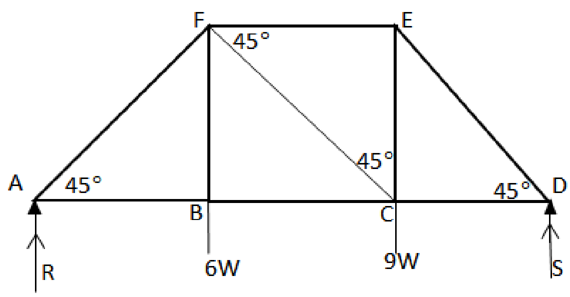

Frameworks

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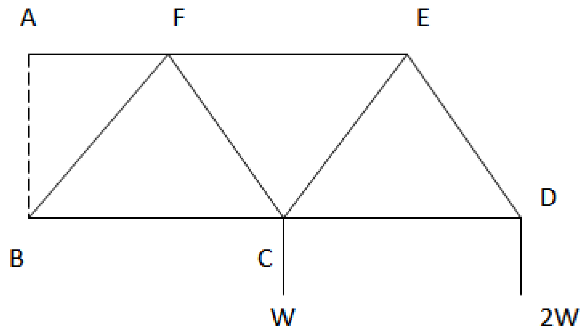

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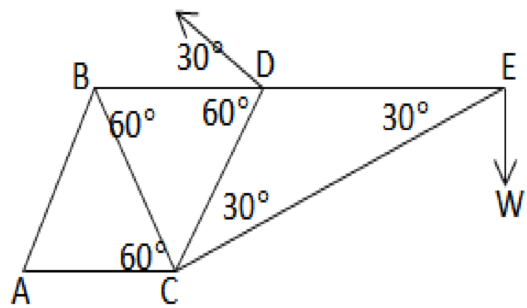

Frameworks

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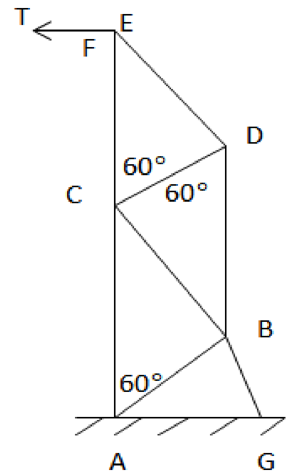

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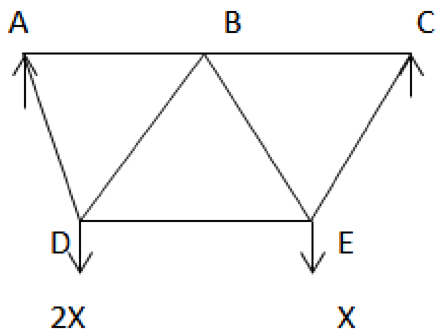

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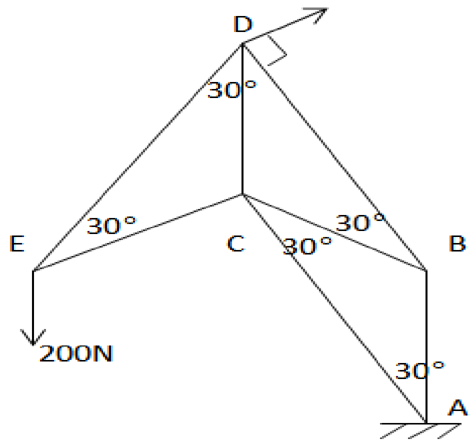

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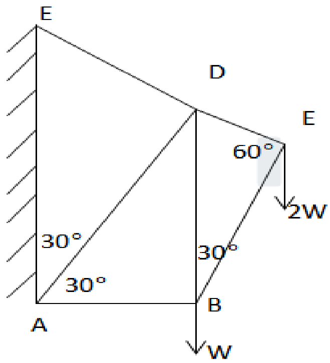

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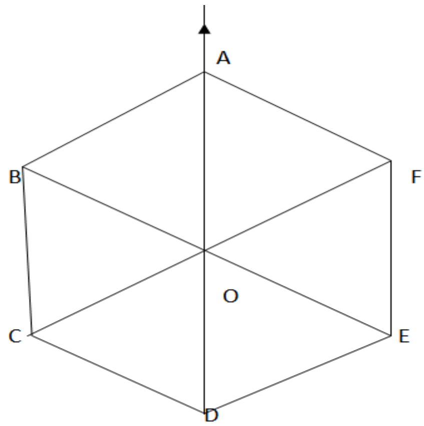
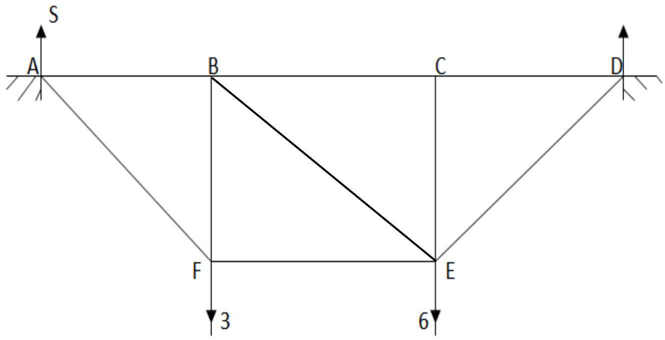


101. The figure shows the girder of a bridge that rests freely on supports at A and D. This framework consists of nine light rods. Six of them, namely AB, BC, CD, BF, CE and FE, are 1 m long and the other three AF, BE and ED are $\sqrt{2}$ m long. As shown in the figure, weights of 3 and 6 metric tons are hung at F and E,

respectively.
Assuming that the reaction of the support D is a vertical force R, find R. Draw a stress diagram by applying bow's notation to the points D, C, E, F, B and A. Hence find the value of S and determine stresses of all the rods and State whether they are thrusts or tensions.



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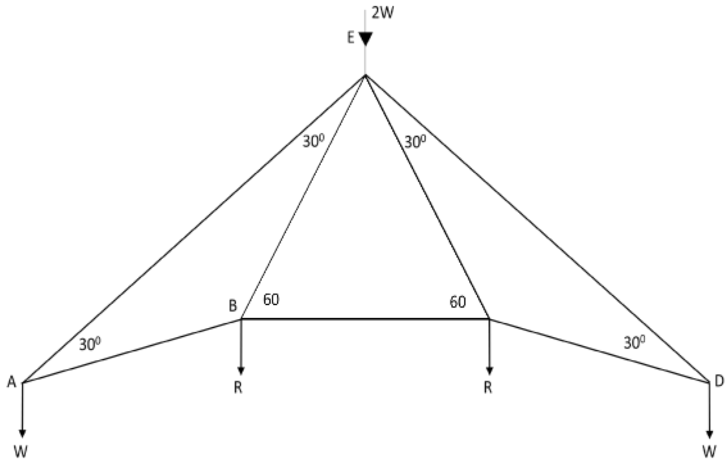
104. The framework made of light rods rests on smooth supports B and C as shown in the figure. BC is horizontal. BCE is an equilateral triangle. ABE and CDE are isosceles triangles. Weights W, W, 2W are applied on A, D, E respectively. The system is in the vertical plane through C. Find the reactions at B and C. Draw a stress diagram and find out the stresses of the rods and whether they are tensions or thrusts.



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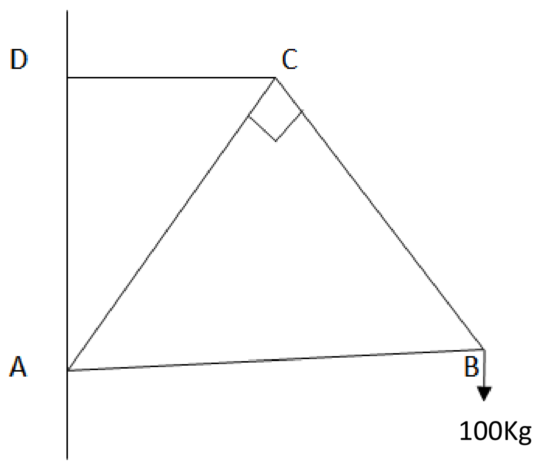

Frameworks

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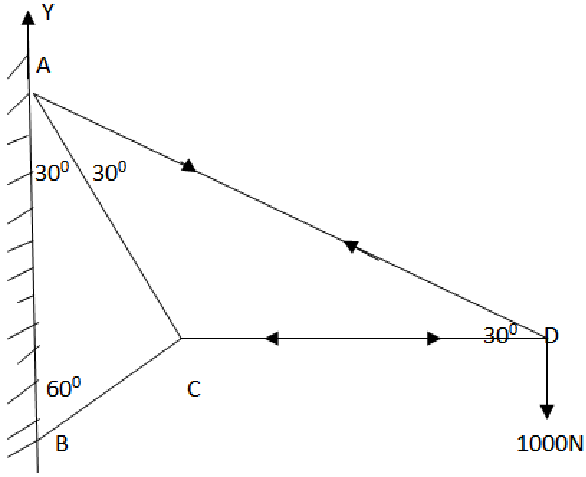

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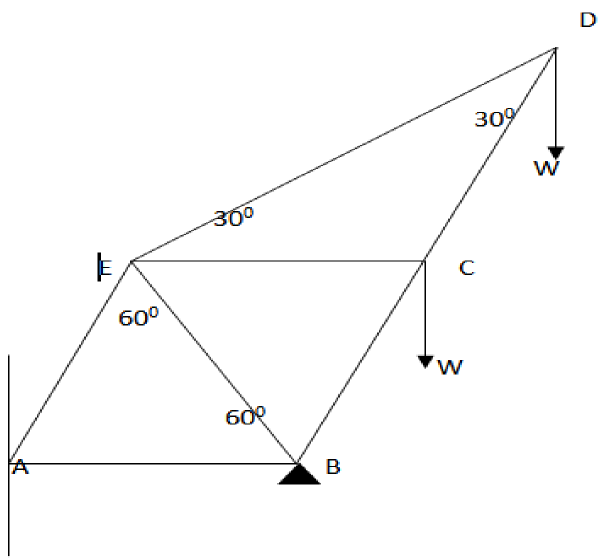
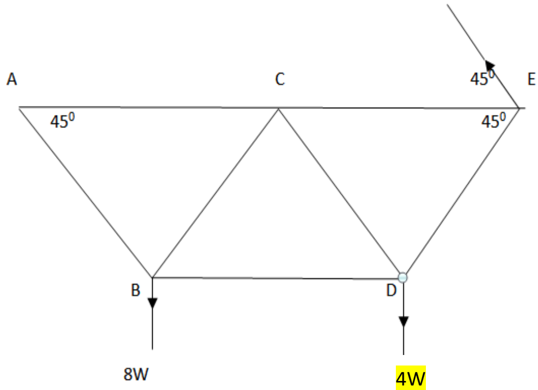


[redacted]

[redacted]

[redacted] The framework shown in the figure is hinged to fixed point A. The system is on the vertical plane through [redacted]



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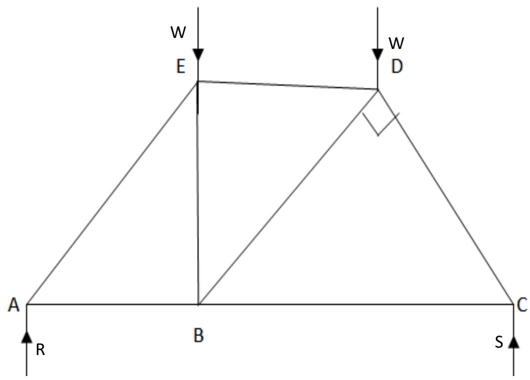
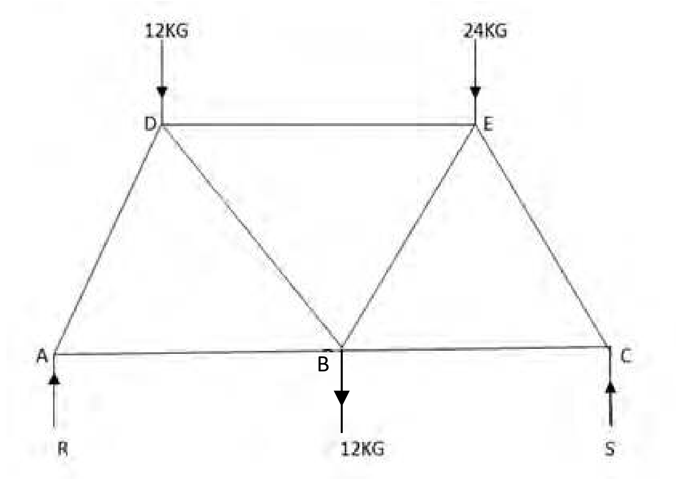
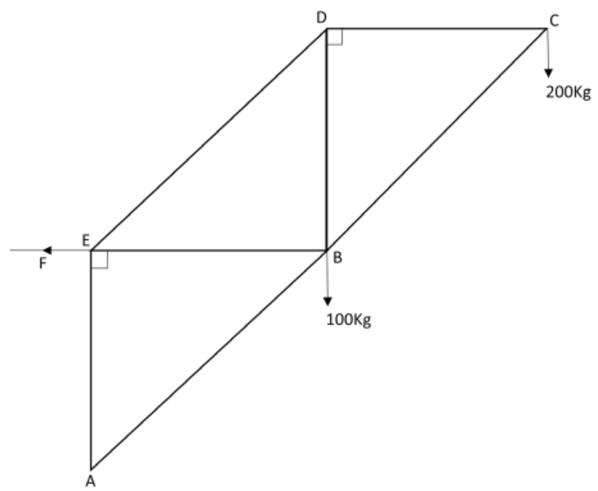


110. Seven light rods of equal length are smoothly hinged at the ends as shown in the figure. The framework is placed vertically in



equilibrium on two supports A and C. AC is horizontal. Weights 24kg, 12kg are applied on E and D respectively. Find R and S considering the equilibrium of external forces. Apply Lami's theorem to the point C and find the stresses of the rods BC and CD and State whether they are tensions or thrusts. Draw a stress diagram for the joints B, D and find the stresses of the rods AB, DE. Find whether they are tensions or thrusts.



111. The framework shown in the figure is made of light rods. Horizontal and vertical rods are equal. The system is smoothly hinged at A. Weights 100kg and 200kg are applied at B and C respectively. The system is in equilibrium in the vertical plane through A by an inextensible horizontal string EF attached to E. Considering the equilibrium of external forces find the tension of the string and the horizontal and vertical components of the reaction at A. Apply Lami's theorem to point C and find the stresses of the rods BC and DC and their conditions. Draw a stress diagram on the section ABDE and find the stresses of the rods BE and AE. State whether they are tensions or thrusts.

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Frameworks

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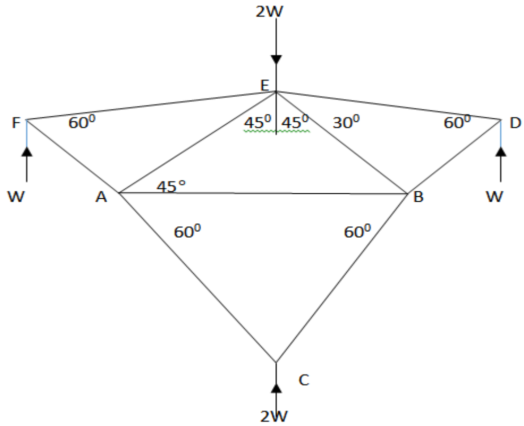

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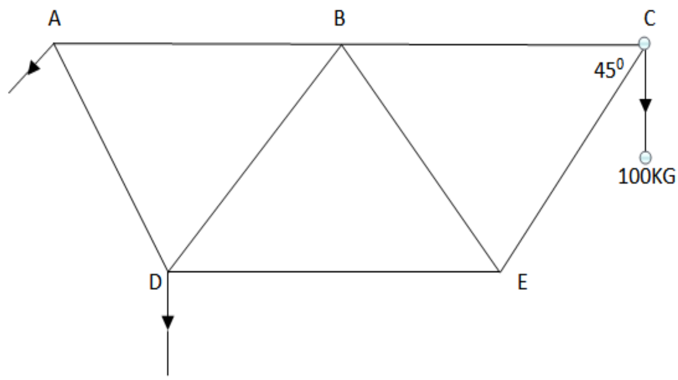

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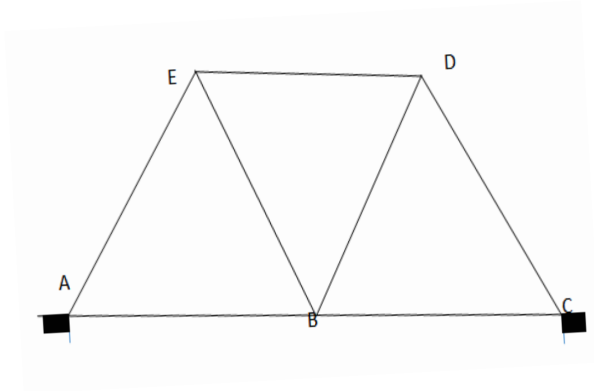

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115. The framework is smoothly hinged at the point A. Horizontal and vertical rods are equal. Weight W is hung at the point D. The system is in equilibrium by a light inelastic string attached to the point B.

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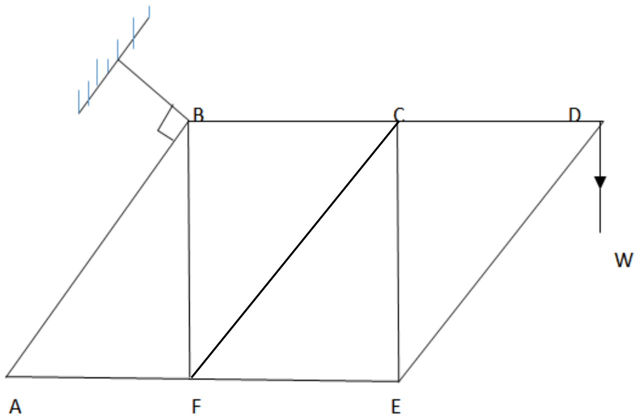

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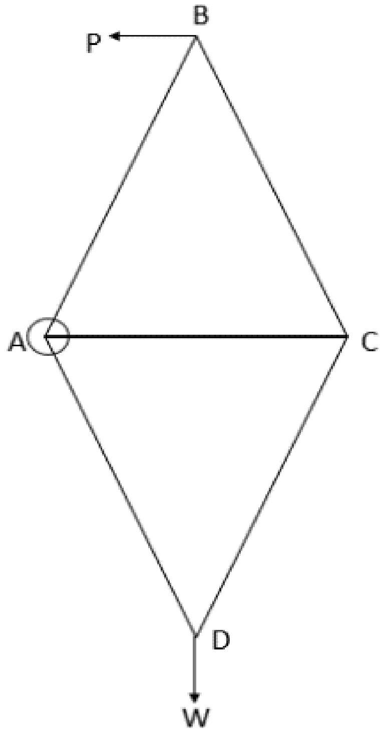

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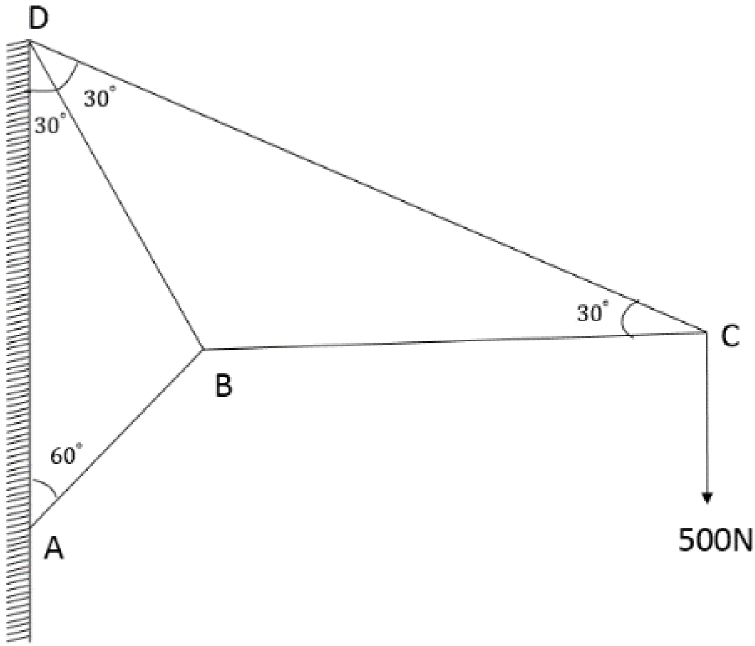

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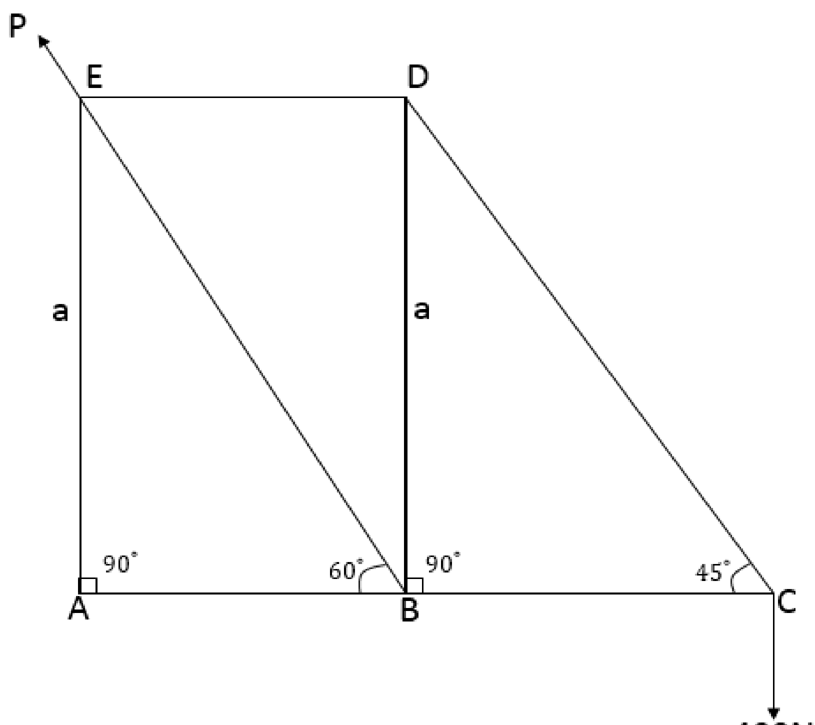

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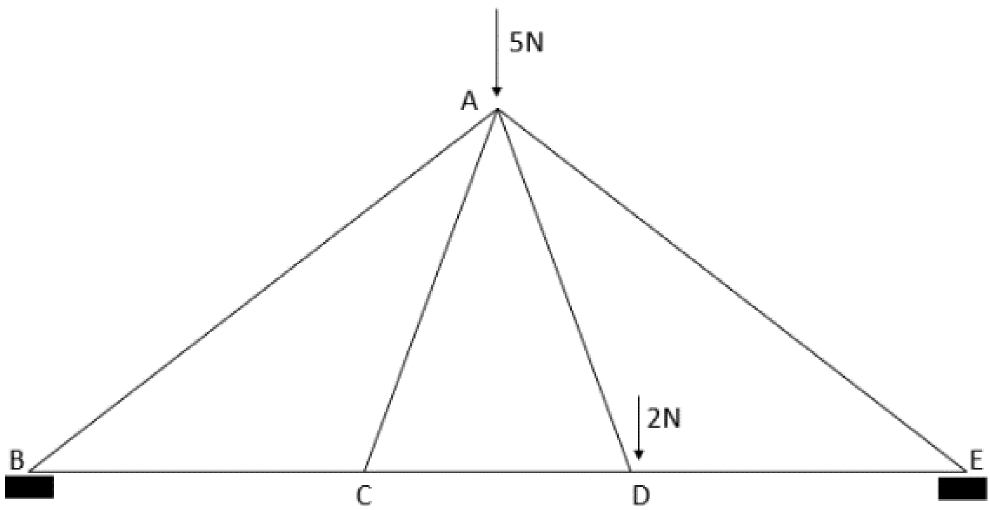

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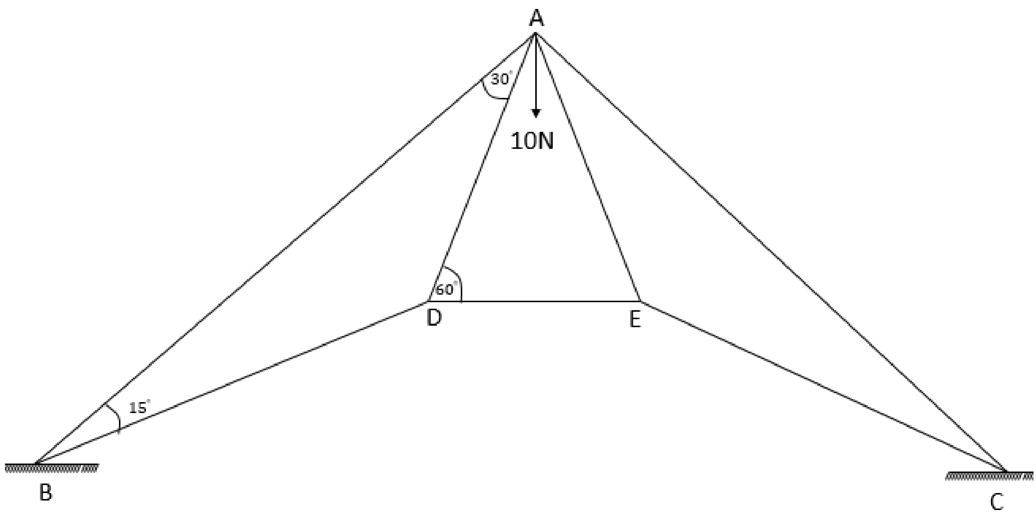


[redacted]

[redacted]

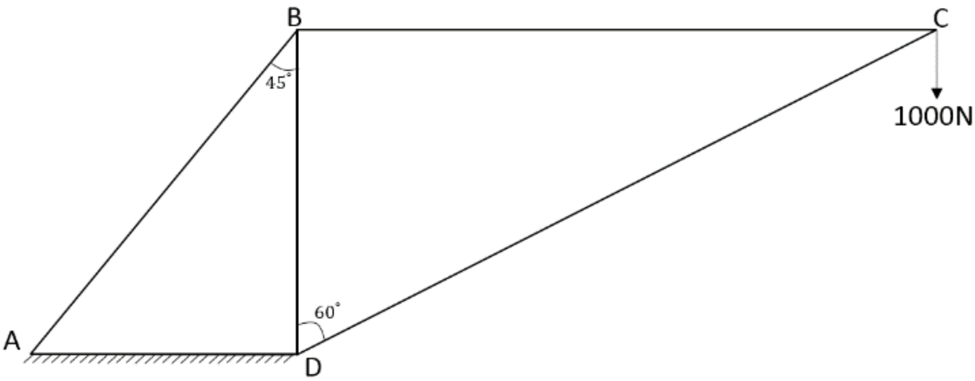
121.

a) AB and BC are two uniform rods of equal length $2a$ and of weights W and 2W respectively. They are smoothly hinged together at B and also hinged at A and C to a fixed horizontal beam. The rods are in equilibrium in a vertical plane with B below AC and $\text{CAB} = \alpha$.

[redacted] Show that the horizontal component of the reaction of the hinge at B is $\frac{3}{4}\ W\ \cot\alpha$, and find the vertical component [redacted]

[redacted]

[redacted]



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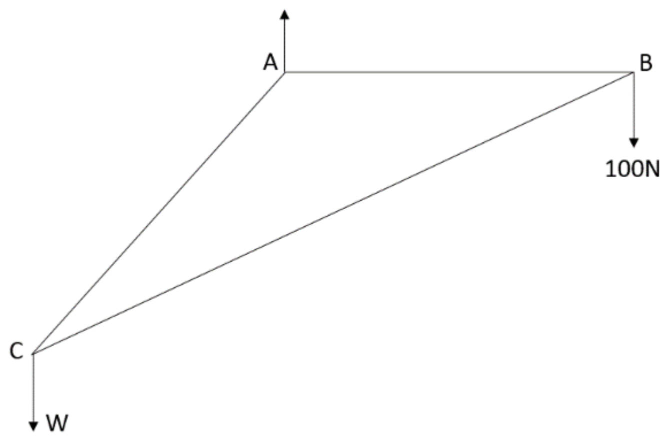


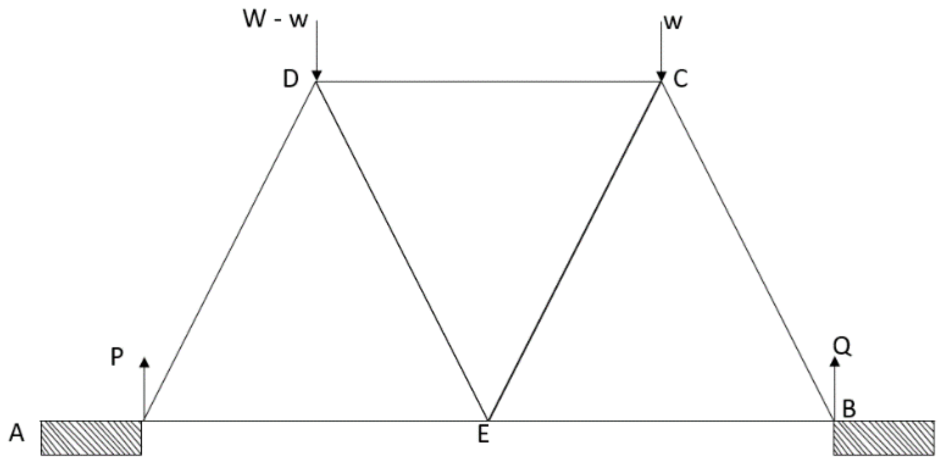
[redacted]

123.

a) A rhombus ABCD is formed of four equal uniform rods. Each of [redacted] maintained by a light rod of length 2a sin α, by connecting mid points of BC and CD. Find the reaction at the joint C.

[redacted]

[redacted]

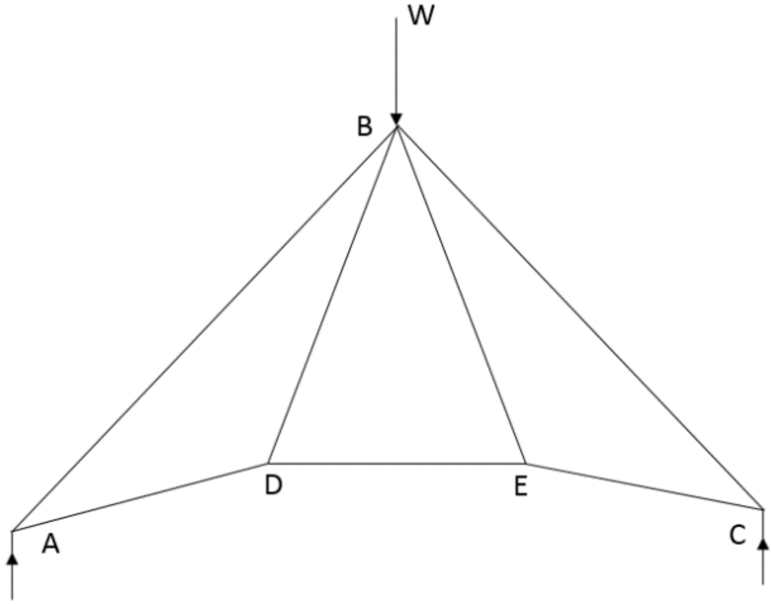


[redacted]

124. a) A rhombus ABCD of side $2a$ consists of four smoothly jointed equal light rods and lies on a smooth horizontal table. The rod AB is fixed. The mid-points of the rods BC and CD are Joined by a light inextensible string which is kept taut by a couple of moment M applied to the rod DA, in the plane of the rhombus. If the angle $A\hat{B}C$ is 2θ, show that

[redacted]

[redacted]



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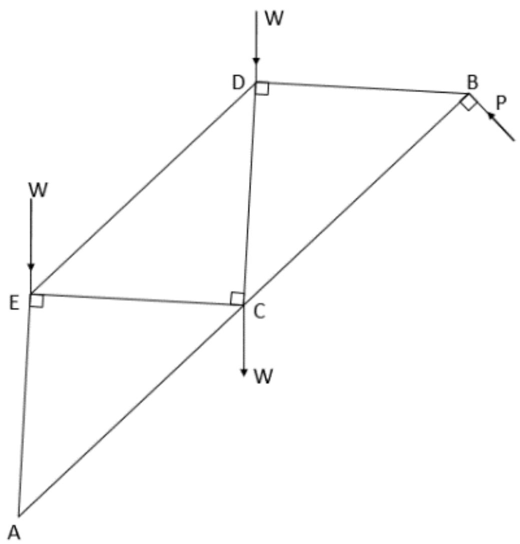

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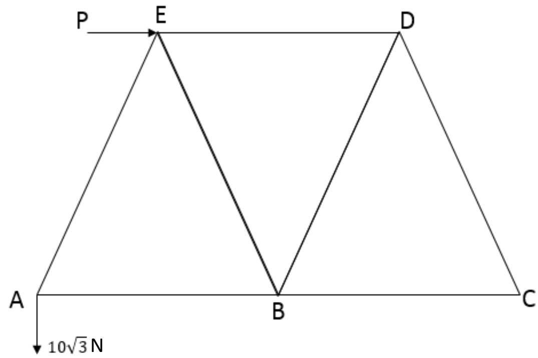


[redacted]

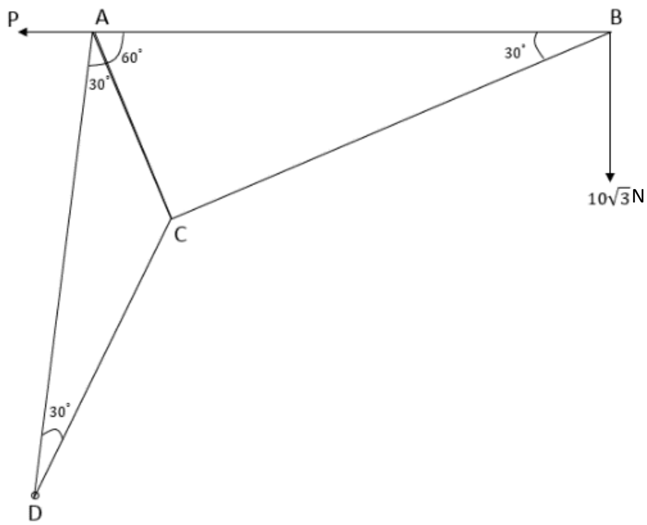
127.

a) Two uniform rods AB and BC are equal in length. The weight of AB is 2w and the weight of BC is w. The rods are smoothly [redacted].

If $A\hat{B}C = 2\theta$, show that the tension of the string is $\frac{3}{2}$w tan θ.

Find the magnitude of the reaction at B and the angle it makes with the horizontal.

[redacted]



[redacted]

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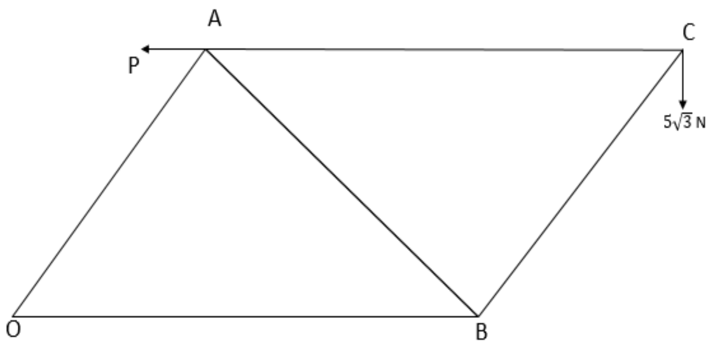

[redacted]

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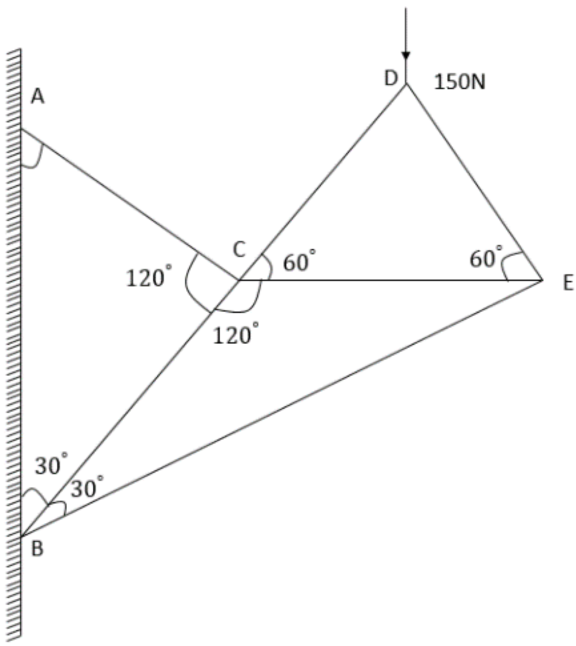

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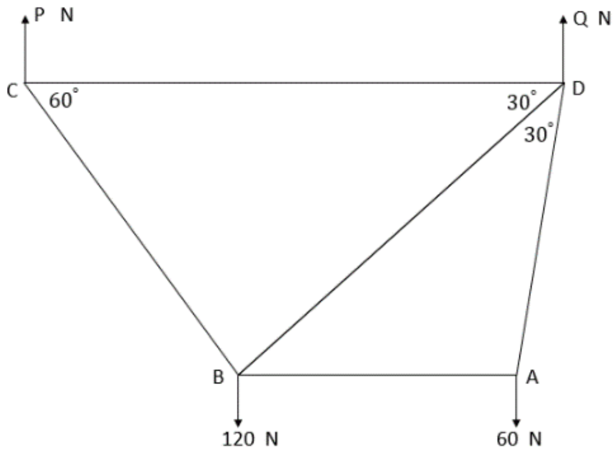
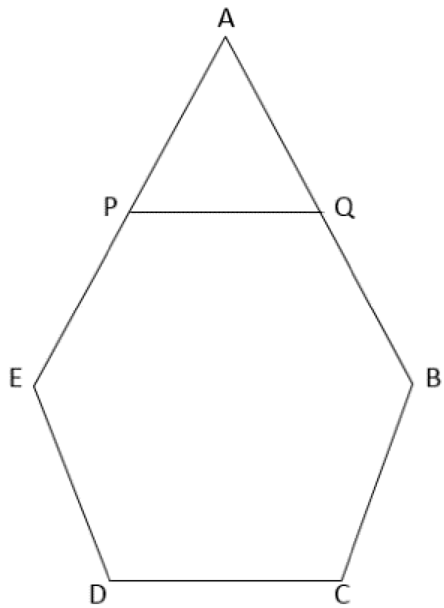


[redacted]

[redacted]

131.

[redacted] Five uniform heavy rods AB, BC, CD, DE and EA are smoothly jointed at their ends to form a framework in the shape of a pentagon ABCDE, as shown in the figure. The rods BC, CD and DE are each of length l and weight W. The rods AB and EA are each of length $2l$ and weight 2w. The two ends P and Q of a light rod PQ of [redacted]



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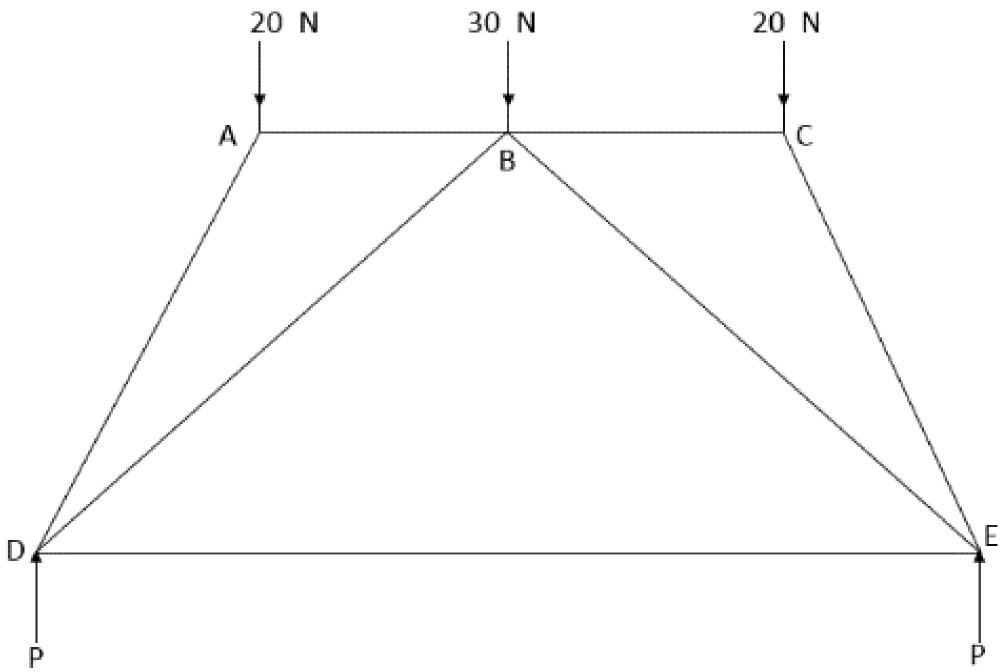


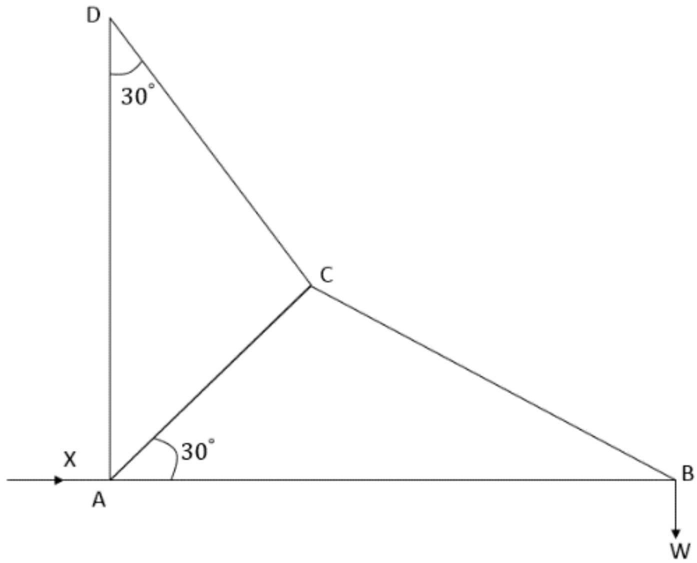
[redacted]

132.

a) Four equal uniform rods, each of weight w_1, are smoothly jointed at their ends to form a rhombus ABCD. The mid-points of BC and [redacted] plane the light rod horizontal. Show that the thrust in light rod is $2(2w_1 + w_2)\tan\theta$.

[redacted]

[redacted]



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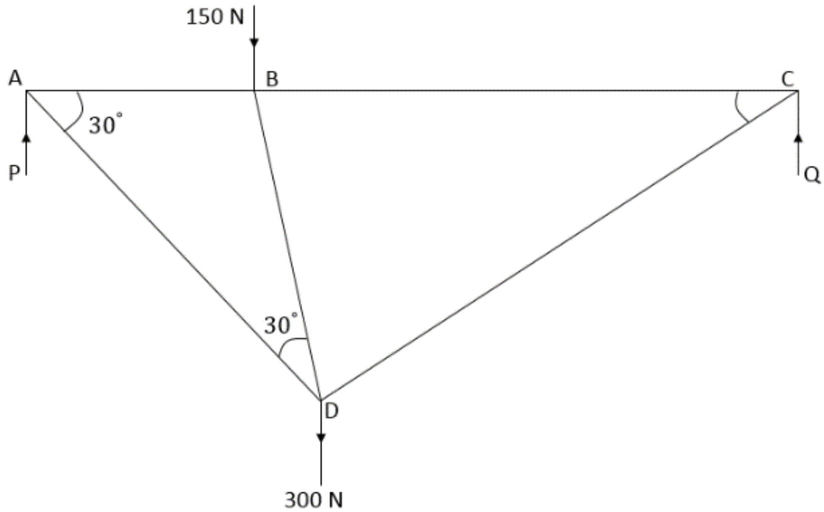
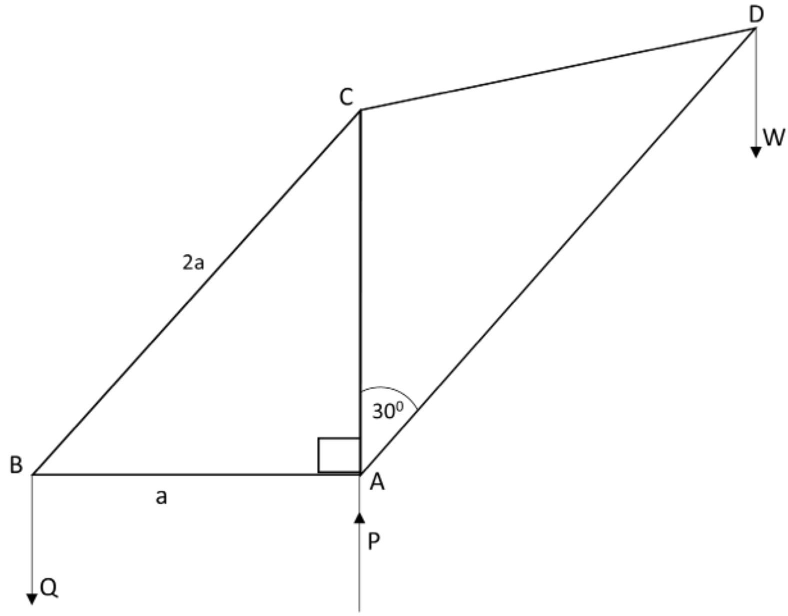


[redacted]

[redacted] The framework shown in the adjoining figure is made or five light rods AB, BC, AC, CD and AD freely jointed at their ends. It is given that AB = a [redacted]



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Raj Wijesinghe
B.Sc. (Math Sp.) M.Sc.