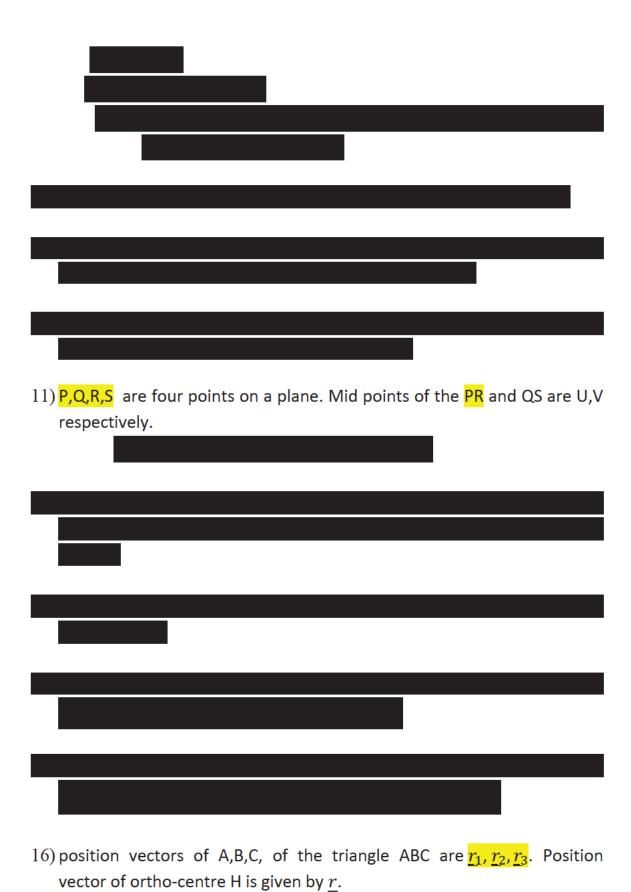
A/L | COMBINED MATHEMATICS

STATICS VECTORS & COPLANAR FORCES



Raj Wijesinghe

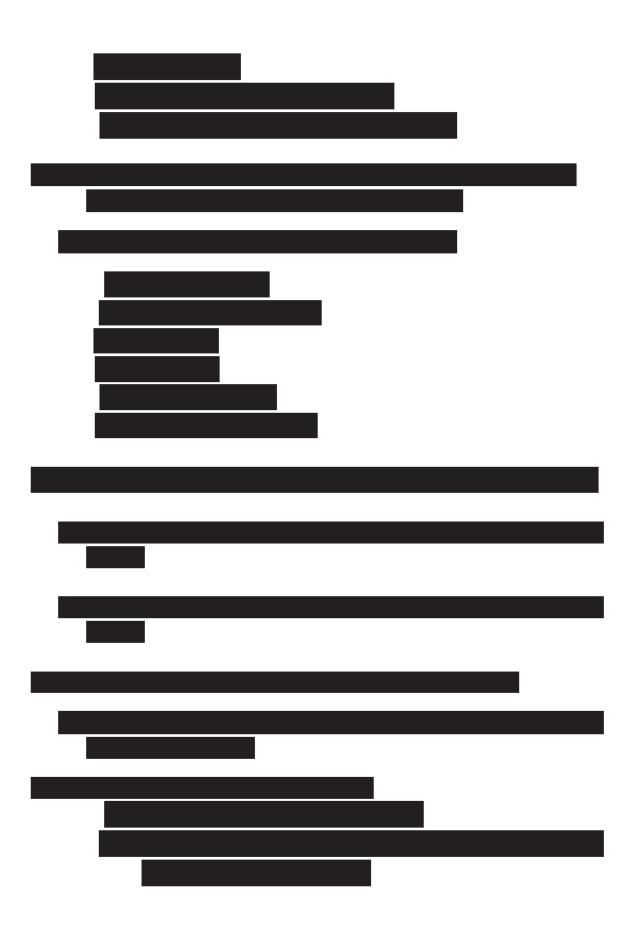


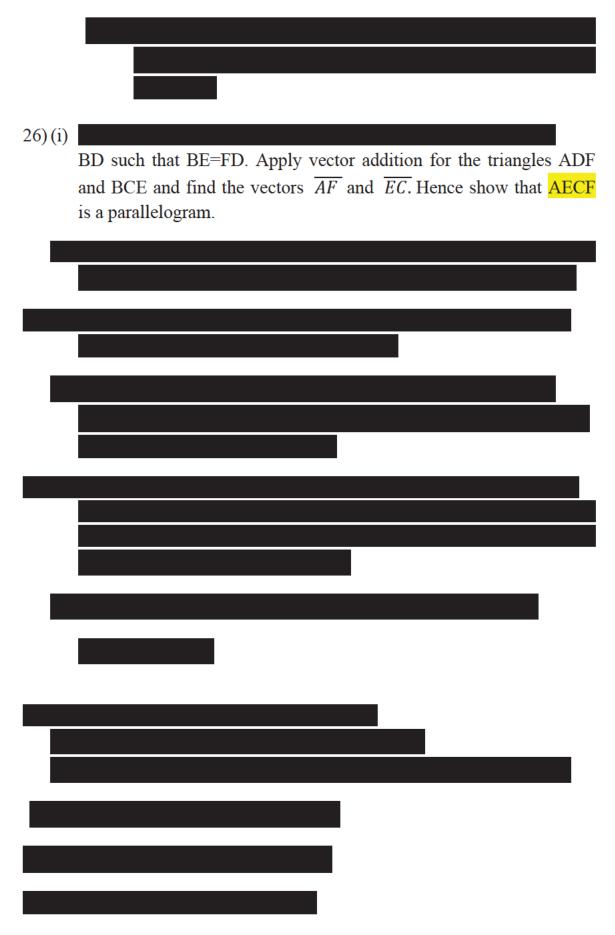


18) The position vector of point A and B are <u>a</u> and <u>b</u>. C is a point on AB such that AC : CB = P : Q . Find the position vector of C and show that it can be written as $\lambda \underline{a} + \mu \underline{b}$, where $\lambda + \mu = 1$ A,B,C,D, are the points on a co - plane. position vector of the points are $\underline{a}, \underline{b}, \underline{c}, \underline{d}$ Respectively, such that $\underline{d} = \lambda \underline{a} + \mu \underline{b} + \gamma \underline{c}$ where $\lambda + \mu + \gamma = 1$. If AB and BC meet at E, Show that the position vector of E is

19) are \underline{a} , \underline{b} , \underline{c} , \underline{d} respectively. Show that $\underline{\alpha}$ + $\underline{\beta}$ + $\underline{\gamma}$ + $\underline{\delta}$ + $\underline{0}$, itence α , β , γ , δ are the scalars such that $\alpha + \beta + \gamma + \delta = 0$ When









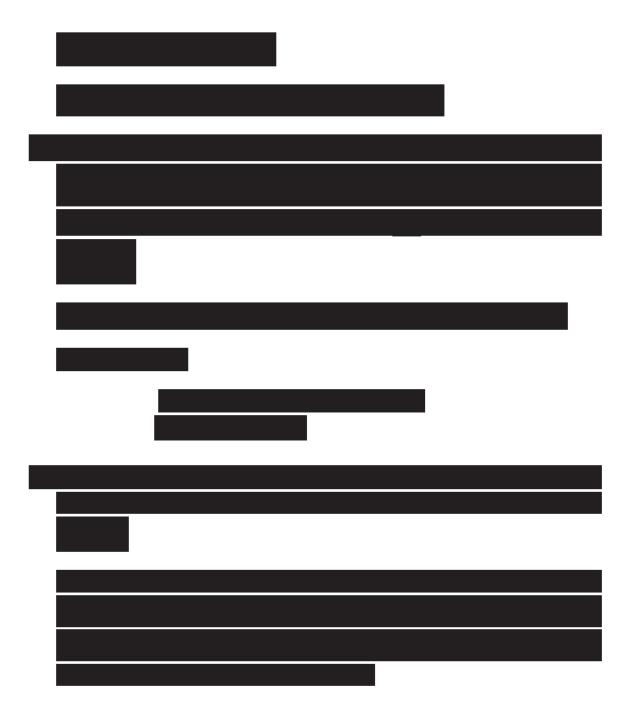
that SG:GP= λ : $(\mu + \nu)$. Show that $\lambda \overrightarrow{GP} + \mu \overrightarrow{GQ} + \nu \overrightarrow{GR} = \overrightarrow{0}$.

Let I be the incentre of the triangle ABC.

Deduce that $\overrightarrow{aIA} + \overrightarrow{bIB} + \overrightarrow{cIC} = 0$

Here a, b, c are the points on the sides BC, CA, AB respectively of the triangle ABC.





40) \underline{a} and \underline{b} are the position vectors of the points A and B relative to the point O which are nonparallel and nonzero. If $\alpha \underline{a} + \beta \underline{b} = 0$, show that $\alpha = 0$ and $\beta = 0$. A, B, O are the non collinear points such that and the angular bisectors of angles AOB and OAB meet at R. Let $a = |\underline{a}|$, b = |b|, $c = |\overline{AB}|$.

 β are nonzero scalars, $\overrightarrow{OA} = a$, $\overrightarrow{OB} = b$, and $\overrightarrow{OC} = \alpha a + \beta b$. Write down \overrightarrow{AB} and \overrightarrow{AC} in terms of α , β , \underline{a} and \underline{b} . If $\alpha + \beta =$ B and C lie on QP and OQ respectively such that $\frac{OB}{BP} = \gamma (> 0)$ and $\frac{QC}{CO} =$ $\mu(>0)$.



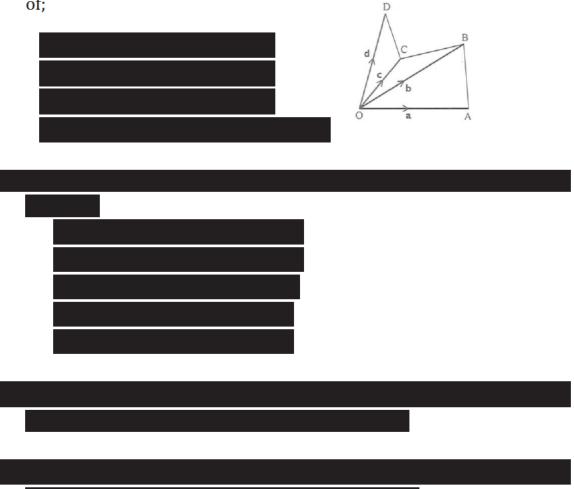
- 47) (i) \underline{a} and \underline{b} are free vectors and the angle between them is θ . Show that $|\underline{a}| |\underline{b}| \le |\underline{a} + |\underline{b}| \le \sqrt{\underline{a^2 + b^2 + 2\underline{a}b\cos\theta}} \le |\underline{a}| + |\underline{b}|$.
 - (ii) O is a point on the plane of triangle ABC. The midpoints of the sides BC, CA, AB are D, E, F respectively.







57) With respect to the origin O, let the position vectors of A, B, C and D be a, b, c and d respectively. In terms of a, b, c and d find the values of;



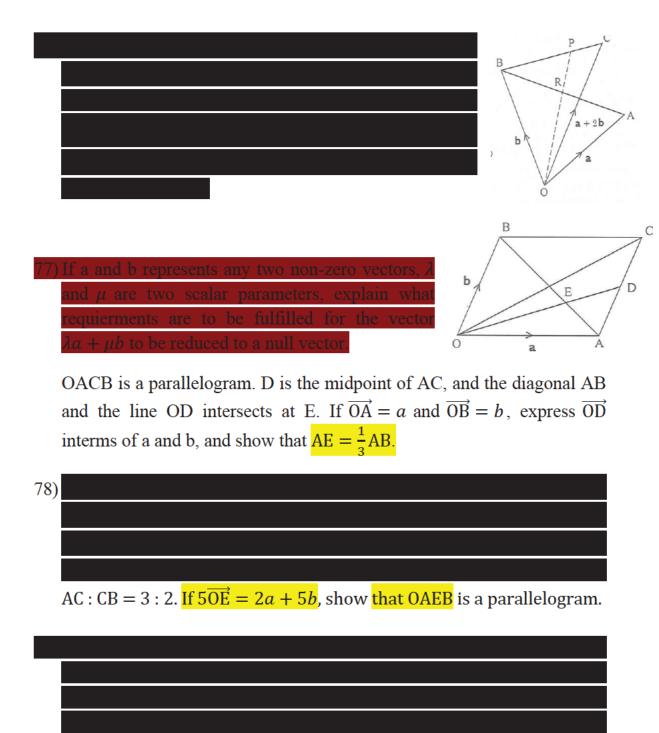
respectively. Show that if $\lambda + \mu \neq 0$, the position vector of P which divides the line AB in the ratio $\lambda : \mu$, is represented by $\frac{\mu a + \lambda b}{\lambda + \mu}$. If $\alpha + \beta + \gamma = 0$ and $\alpha a + \beta b + \gamma c = 0$, prove that the points represented by a, b and c is collinear.

- 64) If a and b represents any two non-zero non-parallel vectors, λ and μ
- 65) OACB is a parallelogram. Let D be the midpoint of AC and, the diagonal AB and the line OD bisect at E. If $\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$, by expressing \overrightarrow{OD} in terms of a and b, show that $\overrightarrow{AE} = \frac{1}{3}\overrightarrow{AB}$.



71) In respect to the origin 0, the position vectors of the points A and B are a = 6i + j and b = 3i + 4j respectively. Find the position vectors of P which divided AB in the ratio 1:2. Hence, find the coordinates of P and $|\overrightarrow{OP}|$.





extended OA, BO and AB respectively such that $\frac{AP}{PO} = \alpha$, $\frac{OQ}{QB} = \beta$ and $\frac{BR}{RA} = \gamma$. Here, $\alpha\beta\gamma \neq 0$. With respect to the origin O, find the position vectors of the points P, Q and R. Hence, if $\alpha\beta\gamma = -1$, show that P, Q

82)

AOB and OAB meet at E. When $|\overrightarrow{OA}| = a$, $|\overrightarrow{OB}| = b$ and |AB| = c and, α and β are scalar, show that $|\overrightarrow{OE}| = \alpha \left\{ \frac{a}{|a|} + \frac{b}{|b|} \right\}$ and $|\overrightarrow{OE}| = a + \beta \left\{ \frac{b}{|c|} - \frac{a}{|c|} - \frac{a}{|a|} \right\}$ and, also show that $\alpha = \frac{ab}{a+b+c}$ and $\beta = \frac{ca}{a+b+c}$. Hence, show that the internal bisectors of the angles of a triangle are concurrent.

83) If $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$ of the parallelogram OACB, prove that $\overrightarrow{BC} = \underline{a}$.







given that a=|a|, b=|b| and c=|a-b|. Show that $\overrightarrow{OP}=\lambda\left\{\frac{a}{|a|}+\frac{b}{|b|}\right\}=a+\mu\left\{\frac{a}{|a|}+\frac{b-a}{|c|}\right\}$. Here λ and μ are scalar. Find λ and μ , hence show that angle OBA is externally bisected by BP.



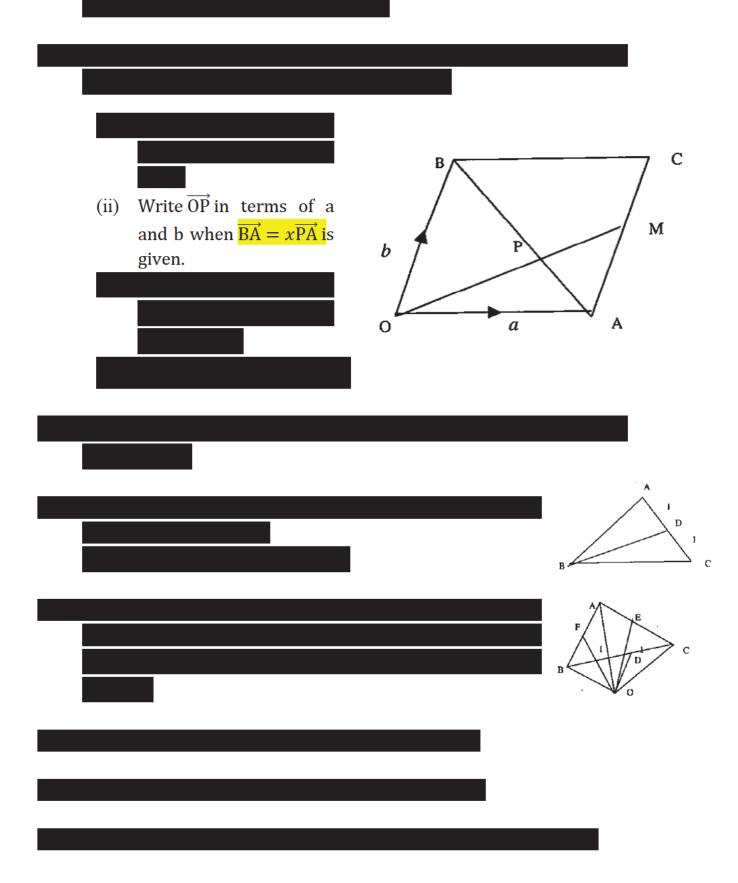
104) P, Q and R are three distinct points. Position vectors of them are $\overrightarrow{OP} = p$, $\overrightarrow{OQ} = q$ and $\overrightarrow{OR} = r$ respectively.

If an a value exists as such r = a p + (1 - a)q, show that P, Q and R are collinear.

Points P, Q and R are located on the sides BC, CA and AB of the triangle ABC such that $\overrightarrow{BP} = \lambda \overrightarrow{PC}$, $\overrightarrow{CQ} = \mu \overrightarrow{QA}$ yd $\overrightarrow{AR} = \nu \overrightarrow{RB}$. Here, $\lambda \mu \nu \neq 0$, $\overrightarrow{CA} = a$ and $\overrightarrow{CB} = b$, find the position vectors of P, Q and R by taking the point C as the origin. Hence, only using $\lambda \mu \nu = -1$, show that P, Q and R are collinear.

	p r 0
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	1 / 1

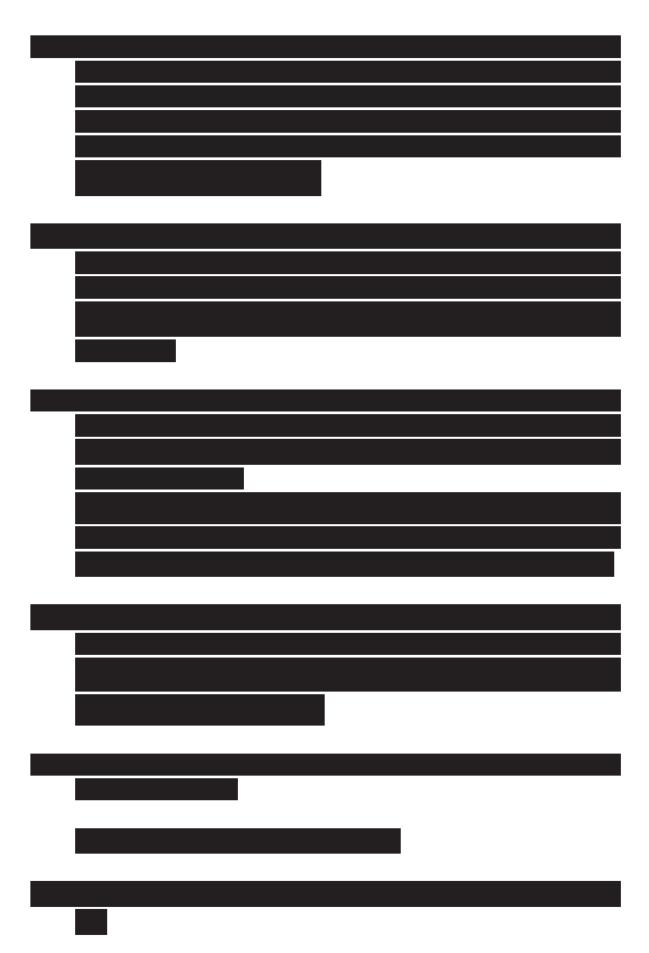
ρ





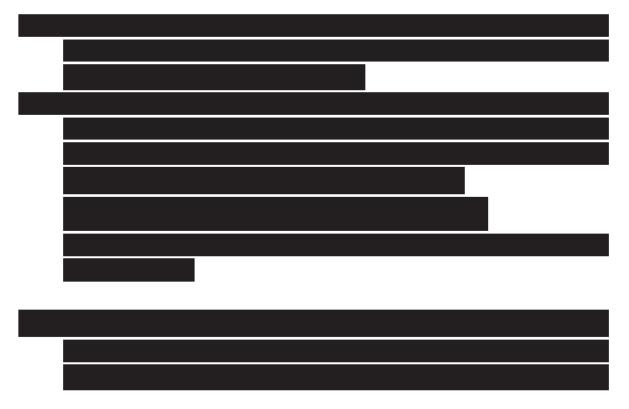
ABCD is a parallelogram O is a point on the plane containing the quadrilateral. Midpoints of the sides AB, BC, CD and DA are E, F, C
127) O, A and B are three collinear points. $\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$.
(i) If $\alpha a + \beta b = 0$, show that $\alpha = 0$ and $\beta = 0$.

- (ii) Show that r = (1 x) a + xb when P is any point on the line AB, $\overrightarrow{OP} = r$ and $0 \le x \le 1$. Hence, show that the diagonals of the parallelogram bisect.





135) In reference with the origin O, the position vectors of the two points A and B are a and b respectively. Show that the position vector of any point on the bisector of the angle AOB can be written as $r = \lambda \left(\frac{a}{|a|} + \frac{b}{|b|}\right)$. Here, λ is a constant parameter. Hence or by using any other method show that $\frac{AL}{LB} = \frac{a}{b}$ if side AB meets the bisector of the angle AOB at L.







- iii. Vector of magnitude 20 units in the direction of \overrightarrow{OP} .
- iv. The angle made by \overline{OP} with the positive direction of the x axis



148) If
$$\underline{a} = 4\underline{i} + 4\underline{j}, \underline{b} = 2\underline{i} + \underline{j}, \underline{c} = 2\underline{i} - 3\underline{j}$$
, find
i) $2\underline{a} + \underline{b} + 3\underline{c}$ ii) $3\underline{a} - 7\underline{b} + 4\underline{c}$

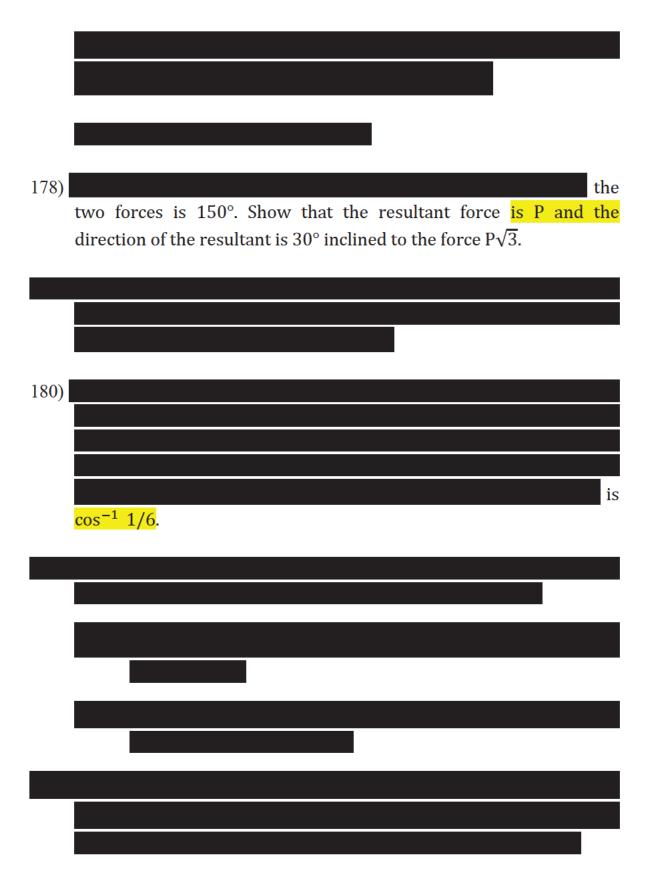


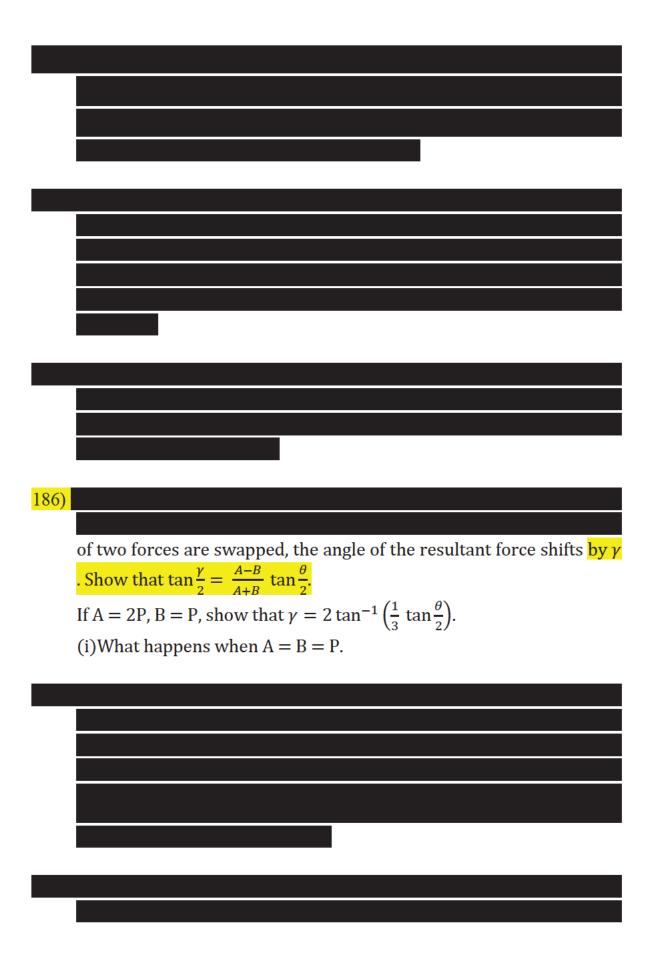




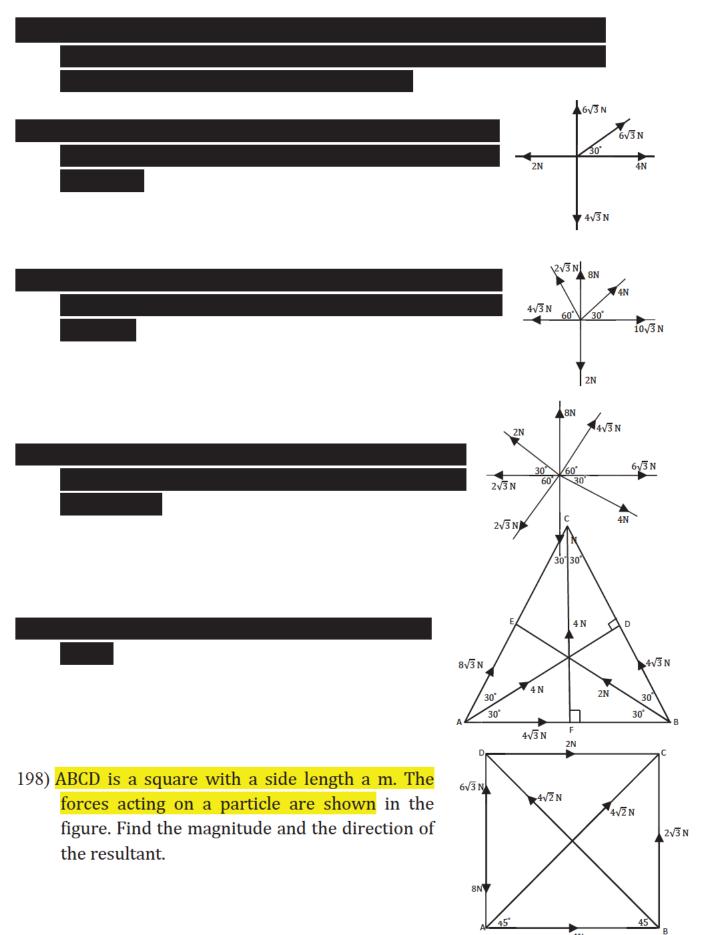


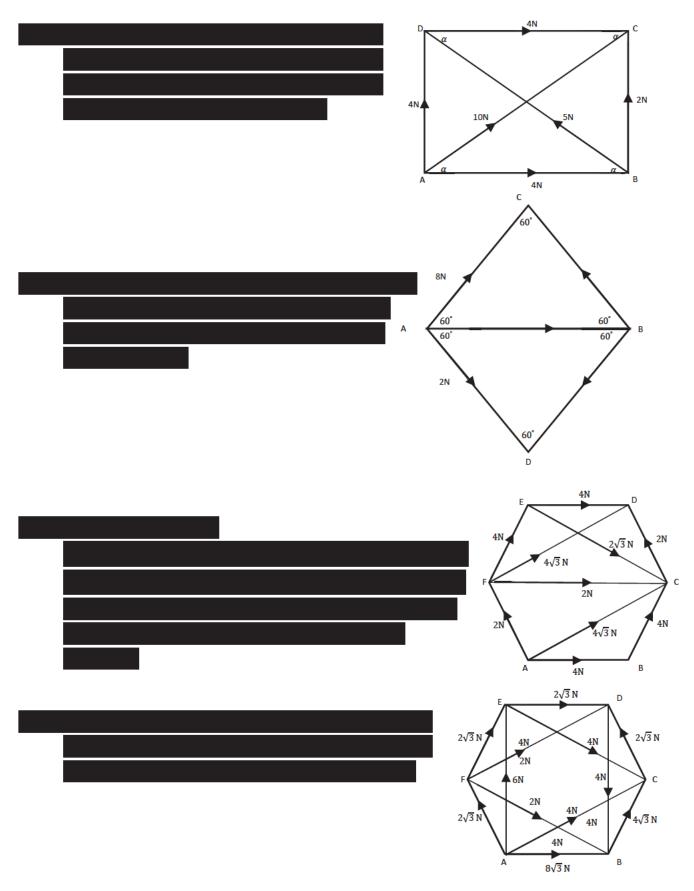






189)	
	x, y. Show that $\alpha = \cos^{-1}\left(\frac{x^2+y^2-A^2-B^2}{2AB}\right)$. When A= P, B= P, and the
	forces are 30° and 60° inclined to the horizontal, find x, y and prove
	that $\alpha = 30^{\circ}$ using the equation.
191)	
	between the first force and the resultant force is $\tan^{-1} \sqrt{3} \frac{(1-\cos\alpha)}{3+\cos\alpha}$.
	Forces of magnitude \sqrt{P} + \sqrt{Q} and \sqrt{P} – \sqrt{Q} act on a particle. The
	resultant of the two forces is $2\sqrt{P-Q}$. Find the angle between the





203) Forces 14P, 6P, $6\sqrt{3}$ P, $4\sqrt{3}$ P, $2\sqrt{3}$ P, 4P, $2\sqrt{3}$ P act on a particle in the directions 60° , 90° , 150° , 210° , 240° , 270° , 300° respectively. Find the magnitude and the direction of the resultant.



	(ii)
	\underline{P} and \underline{Q} along with their magnitudes and directions, \overline{P}
	equivalent to a vertical resultant.
212	
213)	
	and force 2PN.

217)

 P_1 and P_2 that act on a particle. When the angle between P_1 and P_2 is 2α , show that the resultant $is\sqrt{P^2cos^2\alpha+Q^2sin^2\alpha}$.

resultant.

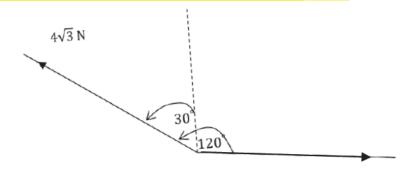
- 218) The angle between P, Q forces acting on a particle is α . The resultant force is $(k+1)\sqrt{P^2+Q^2}$. When the angle between the forces is $90^0-\alpha$, the resultant force is $(k-1)\sqrt{P^2+Q^2}$. Show that the resultant force is $Tan\alpha = \frac{k-2}{k+2}$. Does this true for any value of P, Q. Prove that using when P=4,Q=3.
- 219) The magnitude of the resultant force of the two forces P, Q is P. The magnitude of the resultant of forces 2P, Q acting to the original direction is also P. Show that $Q = \sqrt{3} P$ and the angle between P and Q is 150° .

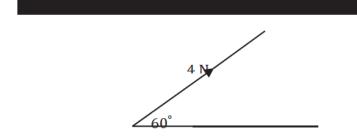
values of the resultant force. If the $\frac{\text{maximum}}{\text{maximum}}$ force is three times the $\frac{\text{minimum force}}{\text{minimum force}}$, find the value of P_2 in terms of P_1 . If the

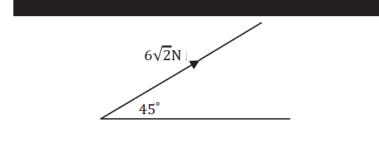
222) Angle between the two forces P, Q acting on a particle is α . The resultant force R inclined θ with the force P. The angle between the forces P+R and Q is α . Show that resultant force of the two forces makes $\theta/2$ with the force P+R

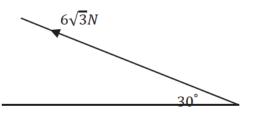
223) Find the vertical and horizontal components.

224) Find the vertical and horizontal components



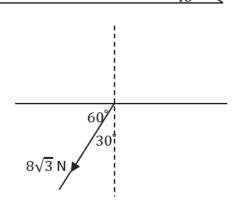


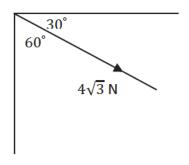


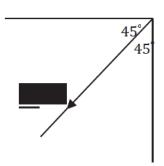


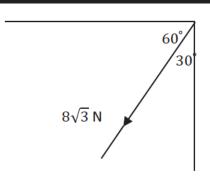
14√2 N

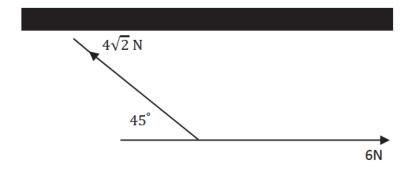




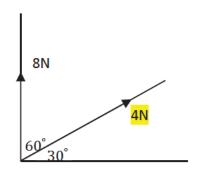


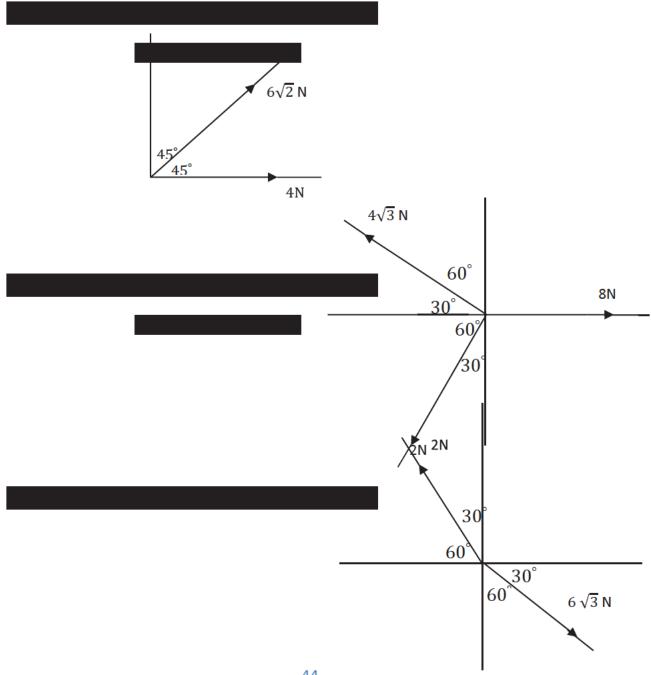


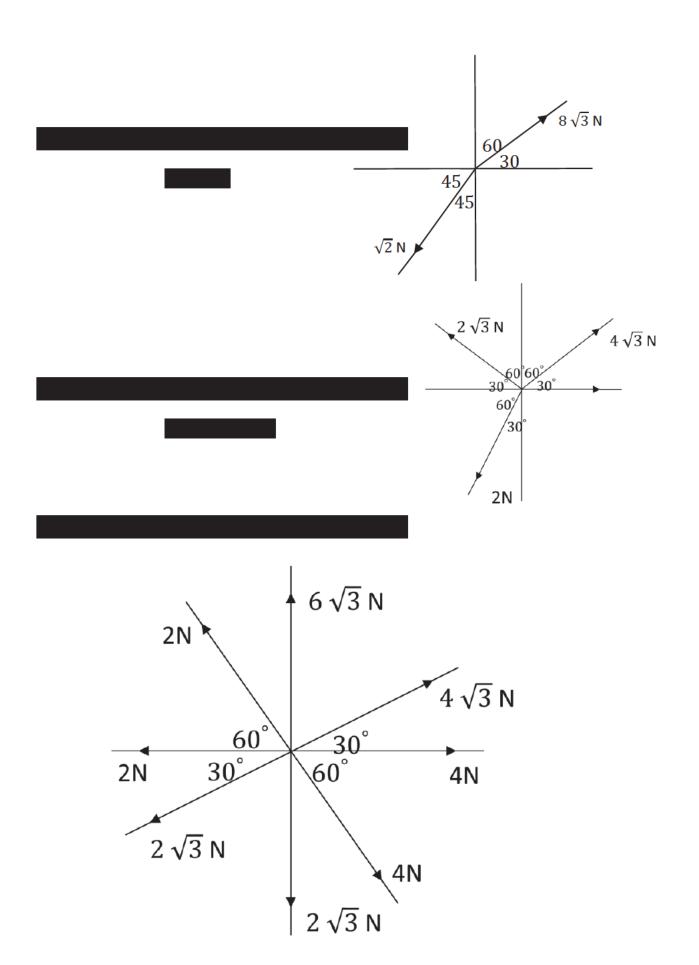




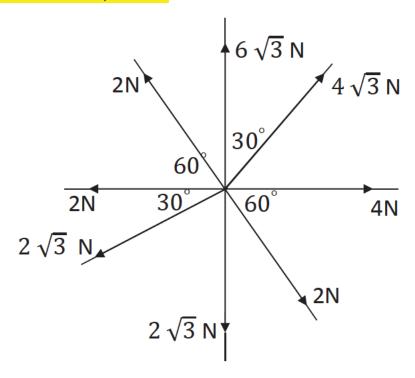
234) Find the vertical and horizontal components



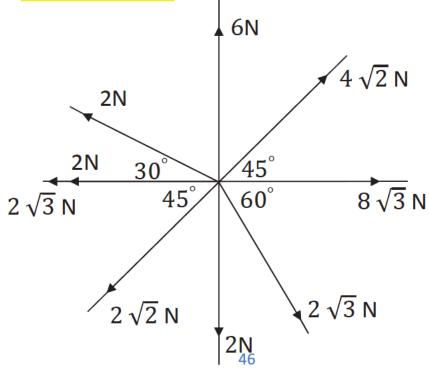


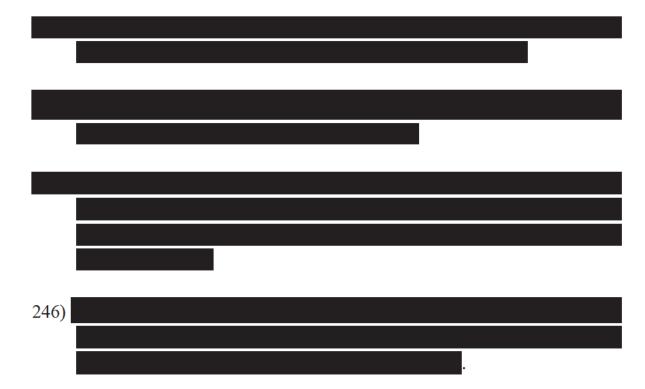


241) Forces acting on a particle is indicated in the diagram. Find the horizontal and vertical components.

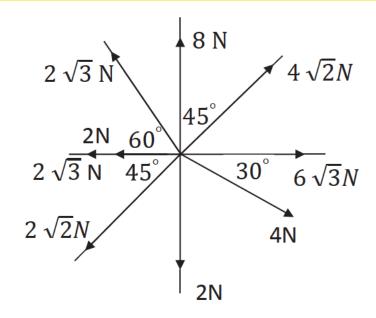


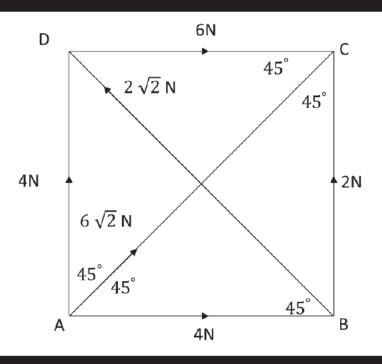
242) Forces acting on a particle is indicated in the diagram. Find the horizontal and vertical components.

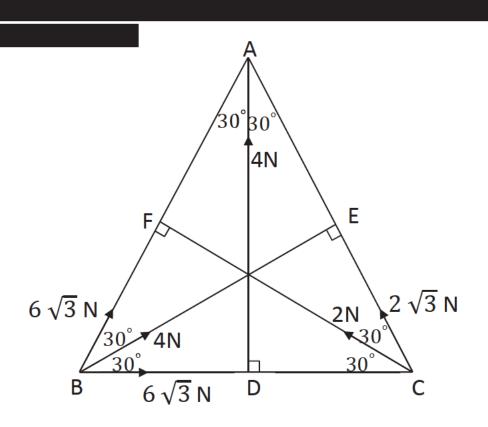


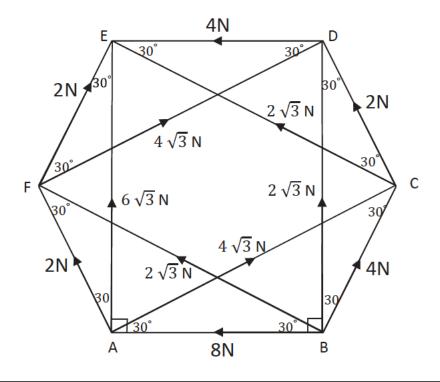


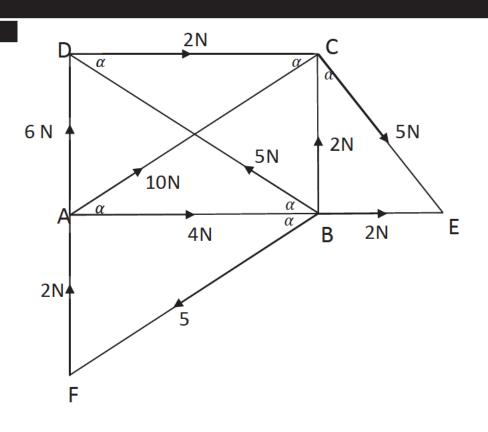
- 247) Forces $5\underline{i} + 2\underline{j}$, $-2\underline{i} + 4\underline{j}$, $-3\underline{i} 5\underline{j}$ and \underline{P} , \underline{Q} act on a particle. The resultant force is $4\underline{i} + 5\underline{j}$. The P and Q forces act in a direction parallel to the vectors $2\underline{i} \underline{j}$ and $\underline{i} + \underline{j}$ respectively. Find the magnitude and the direction of the forces \underline{P} , \underline{Q} .
- The forces acting on a particle is given in the diagram. Find the magnitude and the direction of the resultant. Take the resultant force as R and the inclination of the resultant force to the horizontal as θ .

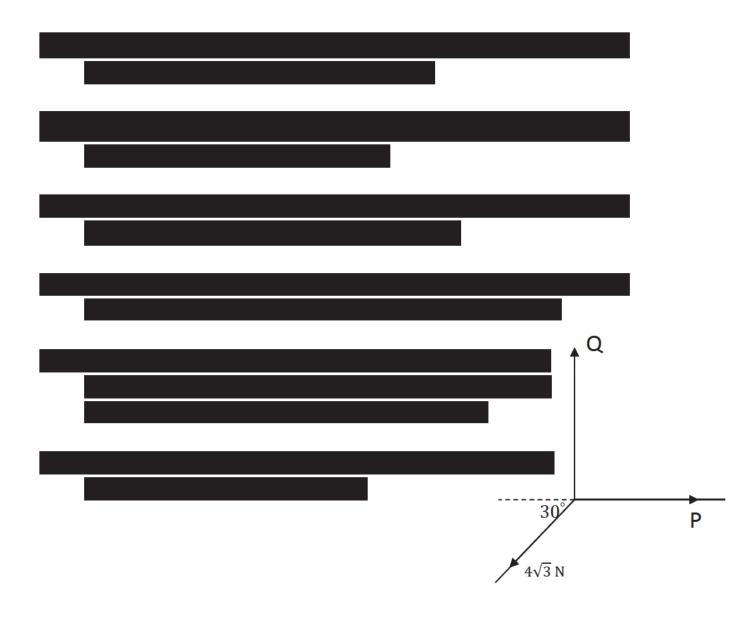


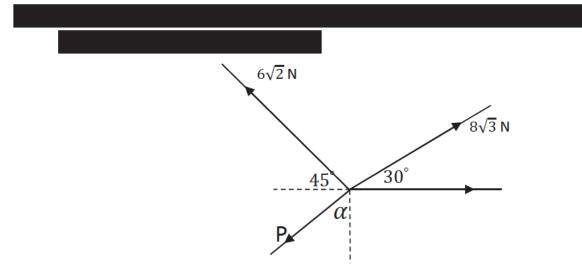




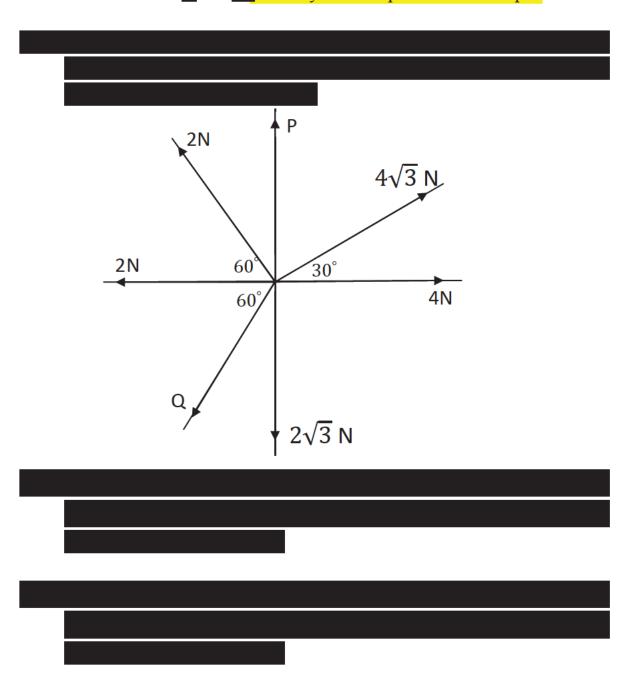


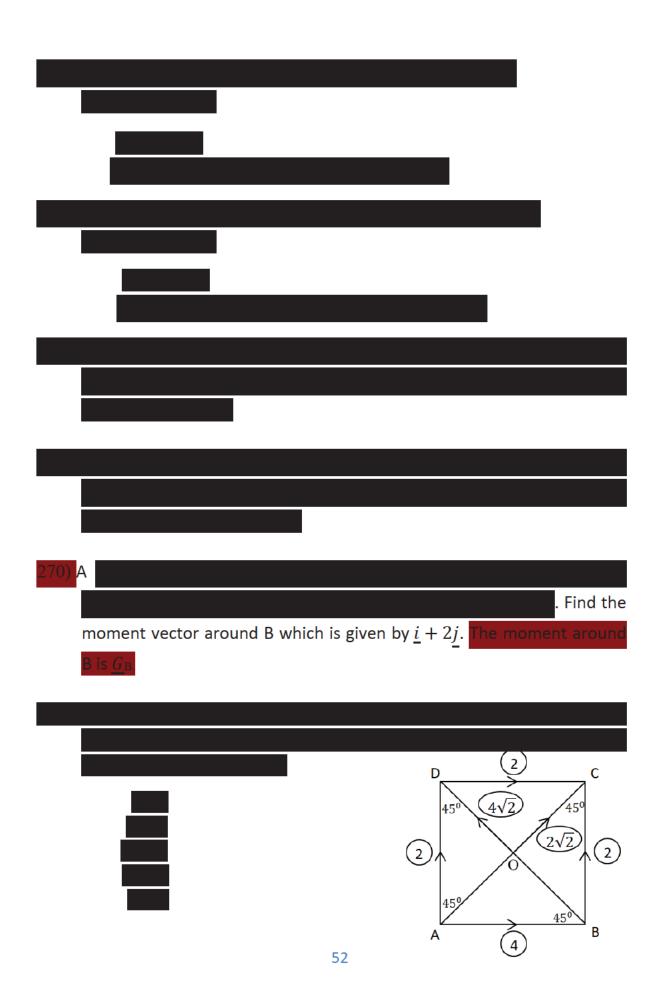




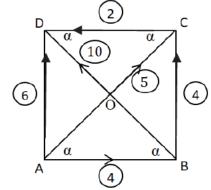


262) Forces $3\underline{i} + \underline{j}$, $4\underline{i} - 3\underline{j}$, $-3\underline{i} - 2\underline{j}$ and \underline{P} , \underline{Q} act on a particle. The forces \underline{P} and \underline{Q} are parallel to the vectors $-2\underline{i} + 3\underline{j}$ and $\underline{i} - 3\underline{j}$. Find the magnitude and the direction of \underline{P} and \underline{Q} , if the system is equivalent to a couple.

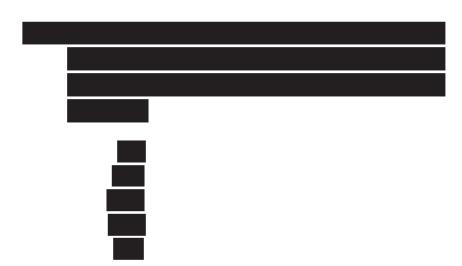


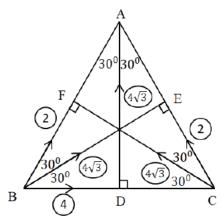


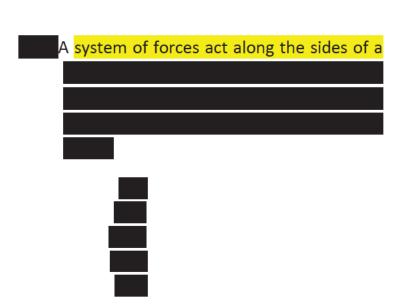
272) A system of forces act along the sides of a square ABCD on a rigid body as shown in the diagram. The length of a side is 2m. AB=4m and BC=3m. Find the resultant moment around the points,

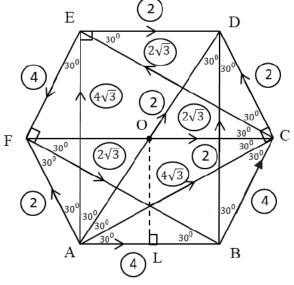


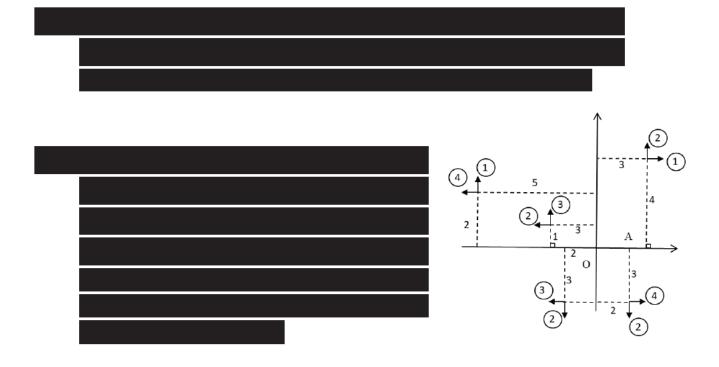












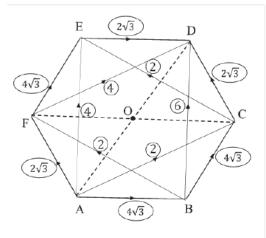
277)

of a side is 2m. Find the magnitude, direction and the line of action of the resultant force. The magnitude of the resultant force is R and the direction is θ to the horizontal. Lets take the distance from A to the

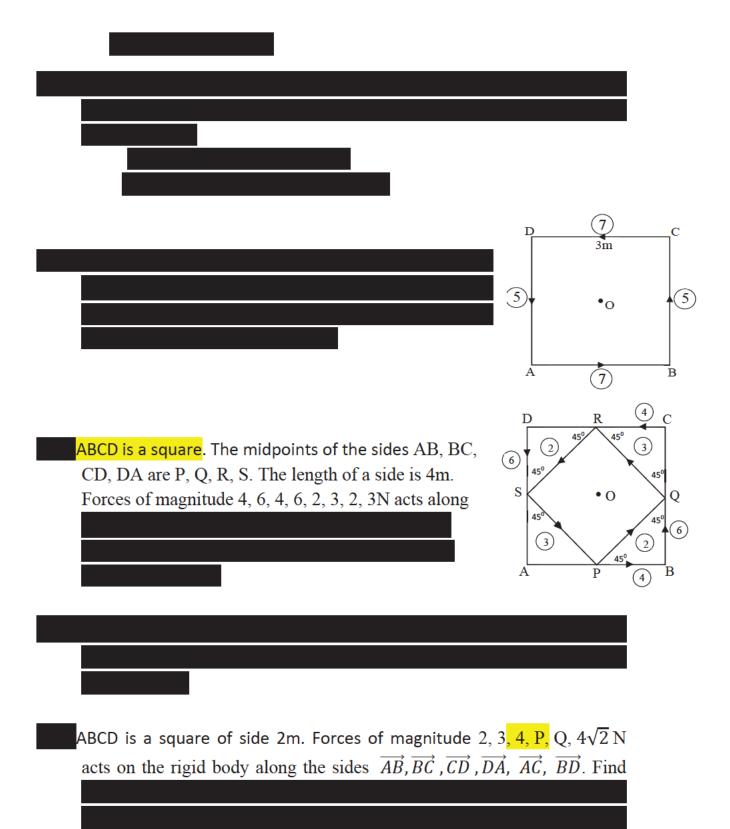
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point where the resultant cuts the AB line as κ m. Find the value of κ .

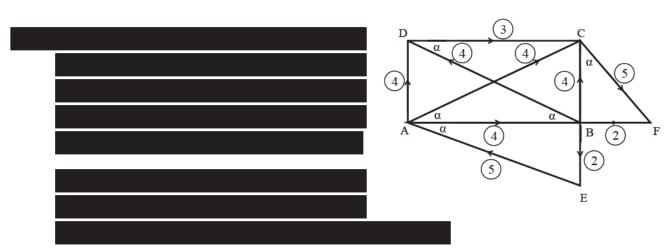
278) ABCDEF is a regular hexagon with am side length. A system of forces act on a rigid body as shown in the figure. Find the magnitude, direction of the resultant force and the distance from A to the point where the resultant force cuts the AB line.



280) magnitude and direction. Find the point where the resultant cuts the OY axis. (Take the horizontal and vertical components of the resultant as X, uts the AB, BC sides. AB = 4m, BC = 3m. The system of forces is ne positions of P. Q and L. 282) Forces $\underline{i} + 3j$ and 3i + 3j acts on a rigid body. i is the unit vector in the horizontal direction. Find the resultant vector along with its magnitude and direction.



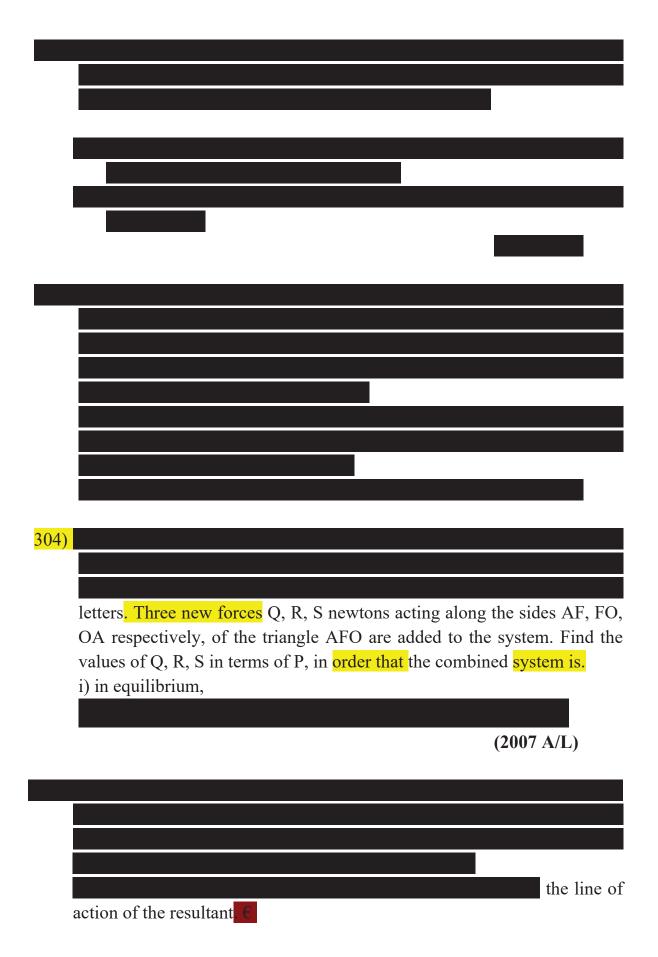
292) ABCDEF is regular hexagon. Forces of magnitude 6P, 2P, P, 7P, P, 2P acting along the sides $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}, \overrightarrow{DE}, \overrightarrow{EF}, \overrightarrow{FA}$ respectively. The length of a side is 2am. Show that system is equivalent to a couple and find its magnitude.

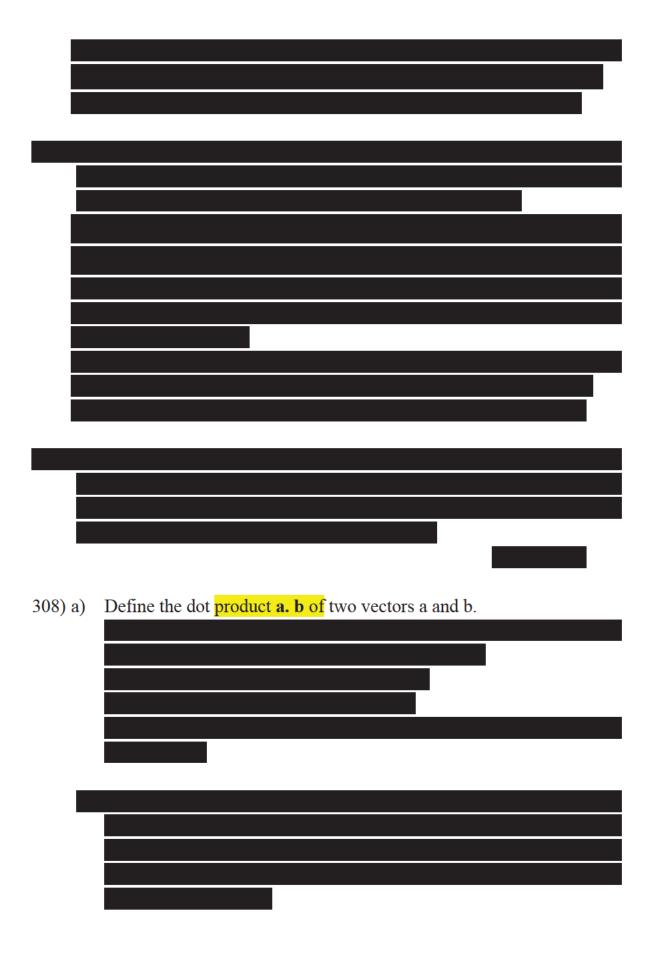


ABC is an equilateral triangle. The perpendicular distance from A to the opposite side is AD. ADCE is a rectangle. AB=2m. Forces of magnitude P, $4, 2, 2, 3\sqrt{3}, 4, 2, Q$ Newton forces act along the sides $\overrightarrow{BA}, \overrightarrow{BD} \overrightarrow{DC}, \overrightarrow{AE}, \overrightarrow{EC}, \overrightarrow{DE}, \overrightarrow{CA}, \overrightarrow{DA}$. When the resultant force R acts

Point	Position Vector	Force
A	2i <mark>+5j</mark>	P(i+3j)
В	4j	-P(2i+ j)
С	-i+ j	P(i -2j)







If the system is in equilibrium, find L, M and N in terms of P. (2011 A/L)

309) $\underline{a} = \underline{i} + \sqrt{3} \underline{j}$ where \underline{i} and \underline{j} have the usual meaning \underline{b} is a vector with magnitude $\sqrt{3}$. If the angle between the vectors \underline{a} and \underline{b} is $\frac{\pi}{3}$, find \underline{b} in the form $x\underline{i} + y\underline{i}$ where x(<0) and y are constants to be determined. (2012 A/L)

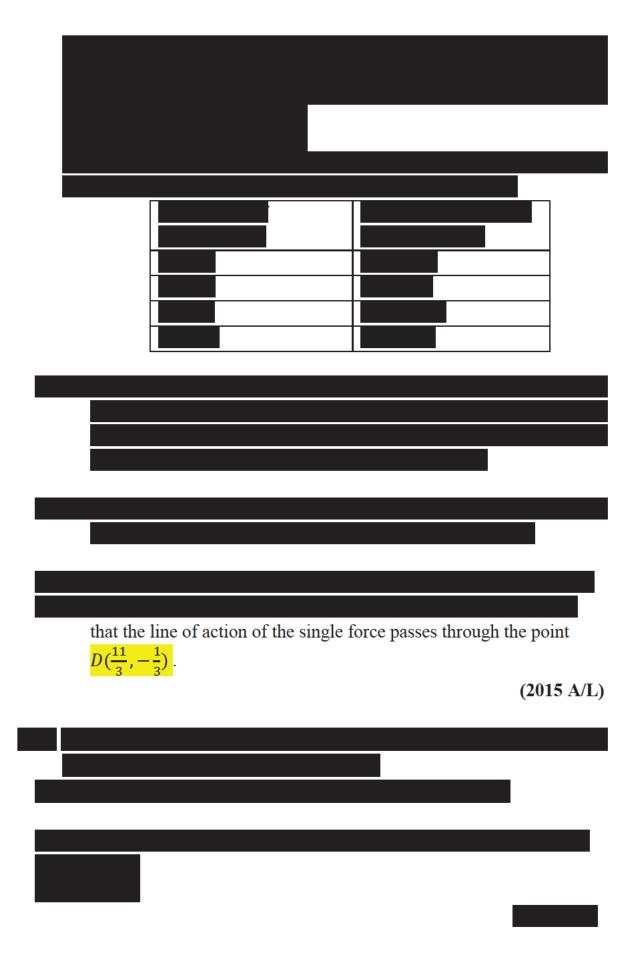


b) The coordinates of the Points A, B and C with respect to a rectangular Cartesian axes Ox and Oy, are $(\sqrt{3},0)(0,-1)$ and $(2\sqrt{3},1)$ respectively.

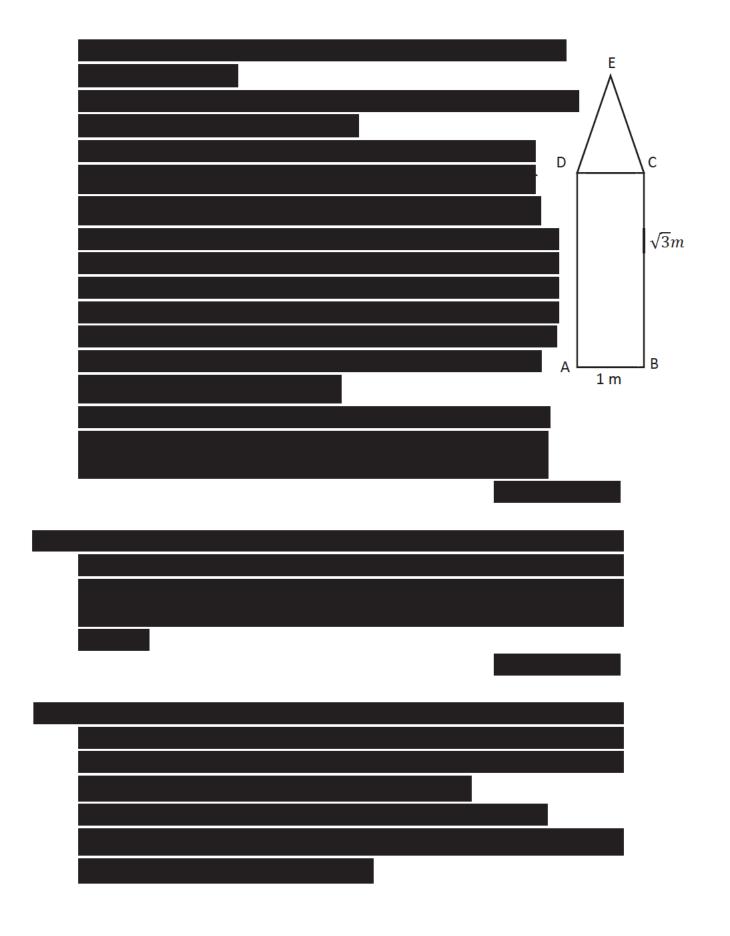


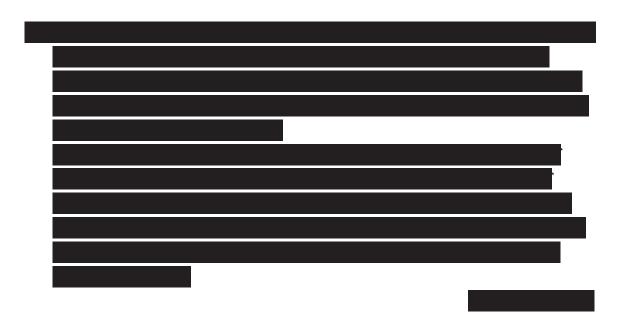
line of action of the resultant is nevelled to DC Also, find the moment
line of action of the resultant is parallel to BC.Also, find the moment the system about O.
If the line of action of the resultant meets AB produced at the point
show that BE $=21$
Now, additional forces of magnitude αP , βP , γP and αP newtons a

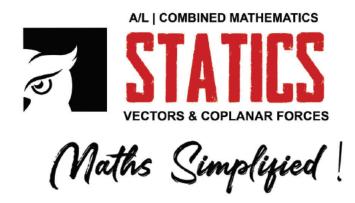




 \overrightarrow{AB} and $\overrightarrow{OQ} = (1 - \lambda) \overrightarrow{OD}$, where $0 < \lambda < 1$. Show that $\overrightarrow{PC} = 2\overrightarrow{CQ}$. (b) In parallelogram ABCD, let AB = 2m and AD = 1m, and let $B\hat{A}D = \frac{\pi}{3}$ Also, let E be the mid-point of CD. Forces of magnitudes 5,5,2,4 and 3 newtons act along AB, BC, DC, DA and BE respectively, in the directions indicated by order of the letters. Show that their resultant force is parallel to AE, and find its magnitude. Also, show that the line of action of the resultant force meets AB produced at a distance $\frac{3}{2}m$ from B. An additional force action through C is now added to the above system of force so that resultant force of the new system is along \overrightarrow{AE} . Find the magnitude and direction of the additional force. (2016 A/L)319) α (>0) is a constant. Using scalar product. Show that $\hat{AOB} = \frac{\pi}{2}$







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