## A/L | COMBINED MATHEMATICS

# STATILS 

## VECTORS \& COPLANAR FORCES



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1)

are the mid points of $\overrightarrow{L M}$ and $\overrightarrow{M N}$ respectively, find LM and OQ .



11) $P, Q, R, S$ are four points on a plane. Mid points of the $P R$ and $Q S$ are $U, V$ respectively.


16) position vectors of $A, B, C$, of the triangle $A B C$ are $\underline{r}_{1}, \underline{r}_{2}, \underline{r}_{3}$. Position vector of ortho-centre H is given by $\underline{r}$.




26) (i)

BD such that $\mathrm{BE}=\mathrm{FD}$. Apply vector addition for the triangles ADF and BCE and find the vectors $\overline{A F}$ and $\overline{E C}$. Hence show that AECF is a parallelogram.







that $\mathrm{SG}: \mathrm{GP}=\lambda:(\mu+v)$. Show that $\lambda \overrightarrow{G P}+\mu \overrightarrow{G Q}+v \overrightarrow{G R}=\overrightarrow{0}$.
Let I be the incentre of the triangle ABC .
Deduce that $a \overrightarrow{\mathrm{I} A}+b \overrightarrow{\mathrm{IB}}+c \overrightarrow{\mathrm{IC}}=0$
Here $\mathrm{a}, \mathrm{b}, \mathrm{c}$ are the points on the sides $\mathrm{BC}, \mathrm{CA}, \mathrm{AB}$ respectively of the triangle ABC .



40) $\underline{a}$ and $\underline{b}$ are the position vectors of the points $A$ and $B$ relative to the point $O$ which are nonparallel and nonzero. If $\alpha \underline{a}+\beta \underline{b}=0$, show that $\alpha=$ 0 and $\beta=0$. $\mathrm{A}, \mathrm{B}, \mathrm{O}$ are the non collinear points such that and the angular bisectors of angles $A O B$ and $O A B$ meet at $R$. Let $\mathrm{a}=|\underline{\mathrm{a}}|, \mathrm{b}=|\mathrm{b}|, \mathrm{c}$ $=\overrightarrow{|A B|}$.


47) (i) $\underline{a}$ and $\underline{b}$ are free vectors and the angle between them is $\theta$. Show that $|\underline{a}|-|\underline{\mathbf{b}}| \leq|\underline{\mathbf{a}}+\underline{\mathrm{b}}| \leq \sqrt{\underline{a^{2}}+b^{2}+2 \underline{a} b \cos \theta} \leq|\underline{\mathrm{a}}|+|\underline{\mathbf{b}}|$.
(ii) O is a point on the plane of triangle ABC . The midpoints of the sides $B C, C A, A B$ are $D, E, F$ respectively.




57) With respect to the origin $O$, let the position vectors of $A, B, C$ and $D$ be $a, b, c$ and d respectively. In terms of $a, b, c$ and $d$ find the values of;




respectively. Show that if $\lambda+\mu \neq 0$, the position vector of P which divides the line AB in the ratio $\lambda: \mu$, is represented by $\frac{\mu a+\lambda b}{\lambda+\mu}$.
If $\alpha+\beta+\gamma=0$ and $\alpha a+\beta b+\gamma c=0$, prove that the points represented by $\mathrm{a}, \mathrm{b}$ and c is collinear.
64) If $a$ and $b$ represents any two non-zero non-parallel vectors, $\lambda$ and $\mu$

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$\square$ liagonal AB and the line OD bisect at E . If $\overline{\mathrm{OA}}=a$ and $\overrightarrow{\mathrm{OB}}=b$, by expressing $\overline{O D}$ in terms of $a$ and $b$, show that $A E=\frac{1}{3} A B$


71) In respect to the origin 0 , the position vectors of the points $A$ and $B$ are $\mathrm{a}=6 \mathrm{i}+\mathrm{j}$ and $\mathrm{b}=3 \mathrm{i}+4 \mathrm{j}$ respectively. Find the position vectors of P which divided AB in the ratio 1:2. Hence, find the coordinates of $P$ and $|\overrightarrow{\mathrm{OP}}|$.



OACB is a parallelogram. D is the midpoint of AC , and the diagonal AB and the line OD intersects at E . If $\overrightarrow{\mathrm{OA}}=a$ and $\overrightarrow{\mathrm{OB}}=b$, express $\overrightarrow{\mathrm{OD}}$ interms of $a$ and $b$, and show that $A E=\frac{1}{3} A B$.
78)

$\mathrm{AC}: \mathrm{CB}=3: 2$. If $5 \overrightarrow{\mathrm{OE}}=2 a+5 b$, show that OAEB is a parallelogram.
$\square$
extended $\mathrm{OA}, \mathrm{BO}$ and AB respectively such that $\frac{A P}{P O}=\alpha, \frac{O Q}{Q B}=\beta$ and $\frac{B R}{R A}=\gamma$. Here, $\alpha \beta \gamma \neq 0$. With respect to the origin 0 , find the position vectors of the points $\mathrm{P}, \mathrm{Q}$ and R . Hence, if $\alpha \beta \gamma=-1$, show that $\mathrm{P}, \mathrm{Q}$


82)

$A O B$ and OAB meet at E. When $|\overrightarrow{\mathrm{OA}}|=a,|\overrightarrow{\mathrm{OB}}|=b$ and $|\mathrm{AB}|=c$ and, $\alpha$ and $\beta$ are scalar, show that $\overrightarrow{O E}=\alpha\left\{\frac{a}{|a|}+\frac{b}{|b|}\right\}$ and $\overrightarrow{O E}=a+$
$\beta\left\{\frac{b}{|c|}-\frac{a}{|c|}-\frac{a}{|a|}\right\}$ and, also show that $\alpha=\frac{a b}{a+b+c}$ and $\beta=\frac{c a}{a+b+c}$.
Hence, show that the internal bisectors of the angles of a triangle are concurrent.
83) If $\overrightarrow{\mathrm{OA}}=\underline{a}$ and $\overrightarrow{\mathrm{OB}}=\underline{b}$ of the parallelogram OACB, prove that $\overrightarrow{B C}=\underline{a}$.




$\square$

given that $a=|a|, b=|b|$ and $c=|\mathrm{a}-\mathrm{b}|$. Show that $\overrightarrow{\mathrm{OP}}=$ $\lambda\left\{\frac{a}{|a|}+\frac{b}{|b|}\right\}=a+\mu\left\{\frac{a}{|a|}+\frac{b-a}{|c|}\right\}$. Here $\lambda$ and $\mu$ are scalar. Find $\lambda$ and $\mu$, hence show that angle OBA is externally bisected by BP.


## 04) $P, Q$ and $R$ are three distinct points. Position vectors of them are $\overrightarrow{O P}=$

$\square$




$A B C D$ is a parallelogram 0 is a point on the plane containing the quadrilateral. Midpoints of the sides $\mathrm{AB}, \mathrm{BC}, \mathrm{CD}$ and DA are $\mathrm{E}, \mathrm{F}, \mathrm{G}$
$\square$

$\square$



135) In reference with the origin 0 , the position vectors of the two points $A$ and B are a and b respectively. Show that the position vector of any point on the bisector of the angle AOB can be written as $r=$ $\lambda\left(\frac{a}{|a|}+\frac{b}{|b|}\right)$. Here, $\lambda$ is a constant parameter. Hence or by using any other method show that $\frac{A L}{L B}=\frac{a}{b}$ if side AB meets the bisector of the angle AOB at L .



iii. Vector of magnitude 20 units in the direction of $\overrightarrow{O P}$.

## The angle made by $O P$ with the positive direction of the $x$ axis


148) If $\underline{a}=4 \underline{i}+4 \underline{j}, \underline{b}=2 \underline{i}+\underline{j}, \underline{c}=2 \underline{i}-3 \underline{j}$, find
i) $2 \underline{a}+\underline{b}+3 \underline{c}$
ii) $3 \underline{a}-7 \underline{b}+4 \underline{c}$






$\square$
$\square$
$\square$
186)
of two forces are swapped, the angle of the resultant force shifts by $\gamma$ . Show that $\tan \frac{\gamma}{2}=\frac{A-B}{A+B} \tan \frac{\theta}{2}$.
If $\mathrm{A}=2 \mathrm{P}, \mathrm{B}=\mathrm{P}$, show that $\gamma=2 \tan ^{-1}\left(\frac{1}{3} \tan \frac{\theta}{2}\right)$.
(i)What happens when $\mathrm{A}=\mathrm{B}=\mathrm{P}$.

189) $\square$
$\mathrm{x}, \mathrm{y}$. Show that $\alpha=\cos ^{-1}\left(\frac{x^{2}+y^{2}-A^{2}-B^{2}}{2 A B}\right)$. When $\mathrm{A}=\mathrm{P}, \mathrm{B}=\mathrm{P}$, and the forces are $30^{\circ}$ and $60^{\circ}$ inclined to the horizontal, find $\mathrm{x}, \mathrm{y}$ and prove that $\alpha=30^{\circ}$ using the equation.
$\square$
191) $\square$
between the first force and the resultant force is $\tan ^{-1} \sqrt{3} \frac{(1-\cos \alpha)}{3+\cos \alpha}$.
$\square$
Forces of magnitude $\sqrt{P}+\sqrt{Q}$ and $\sqrt{\underline{P}}-\sqrt{Q}$ act on a particle. The resultant of the two forces is $2 \sqrt{P-Q}$. Find the angle between the
$\square$


203) Forces $14 \mathrm{P}, 6 \mathrm{P}, 6 \sqrt{3} P, 4 \sqrt{3} P, 2 \sqrt{3} P, 4 P, 2 \sqrt{3} P$ act on a particle in the directions $60^{\circ}, 90^{\circ}, 150^{\circ}, 210^{\circ}, 240^{\circ}, 270^{\circ}, 300^{\circ}$ respectively. Find the magnitude and the direction of the resultant.


(ii)

$\underline{P}$ and $\underline{Q}$ along with their magnitudes and directions, if the system is equivalent to a vertical resultant.

$P_{1}$ and $P_{2}$ that act on a particle. When the angle between $P_{1}$ and $P_{2}$ is $2 \alpha$, show that the resultant is $\sqrt{P^{2} \cos ^{2} \alpha+Q^{2} \sin ^{2} \alpha}$.

## resultant.

218) The angle between $P, Q$ forces acting on a particle is $\alpha$. The resultant force is $(k+1) \sqrt{\mathrm{P}^{2}+\mathrm{Q}^{2}}$. When the angle between the forces is $90^{\circ}-\alpha$, the resultant force is $(k-1) \sqrt{P^{2}+Q^{2}}$. Show that Tan $\alpha=\frac{k-2}{\mathrm{k}+2}$. Does this true for any value of $\mathrm{P}, \mathrm{Q}$. Prove that using when $\mathrm{P}=4, \mathrm{Q}=3$.
219) The magnitude of the resultant force of the two forces $P, Q$ is $P$. The magnitude of the resultant of forces $2 \mathrm{P}, \mathrm{Q}$ acting to the original direction is also P . Show that $\mathrm{Q}=\sqrt{3} \mathrm{P}$ and the angle between P and Q is $150^{\circ}$.

values of the resultant force. If the maximum force is three times the minimum force, find the value of $\mathrm{P}_{2}$ in terms of $\mathrm{P}_{1}$. If the $\square$


## 224) Find the vertical and horizontal components



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241) Forces acting on a particle is indicated in the diagram. Find the horizontal and vertical components.

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247) Forces $5 \underline{i}+2 \dot{i},-2 \underline{i}+4 \dot{j},-3 \underline{i}-5 \dot{i}$ and $\underline{P}, \underline{Q}$ act on a particle. The resultant force is $4 \underline{i}+5 \dot{j}$. The P and Q forces act in a direction parallel to the vectors $2 \underline{i}-\dot{i}$ and $\underline{i}+i$ respectively. Find the magnitude and the direction of the forces $\underline{P}, \underline{Q}$.
248) The forces acting on a particle is given in the diagram. Find the magnitude and the direction of the resultant. Take the resultant force as $R$ and the inclination of the resultant force to the horizontal as $\theta$.





262) Forces $3 \underline{i}+\underline{i}, 4 \underline{i}-3 \dot{j},-3 \underline{i}-2$ ind $\underline{P}, \underline{Q}$ act on a particle. The forces $\underline{P}$ and Q are parallel to the vectors $-2 \underline{i}+3 \dot{j}$ and $\underline{i}-3 \mathrm{j}$. Find the magnitude and the direction of $\underline{\mathrm{P}}$ and $\underline{\mathrm{O}}$. if the system is equivalent to a couple.




Find the moment vector around B which is given by $\underline{i}+2 \underline{j}$. The moment around 3 is $G_{B}$

272) A system of forces act along the sides of a square $A B C D$ on a rigid body as shown in the diagram. The length of a side is $2 m$. $A B=4 m$ and $B C=3 m$. Find the resultant moment around the points,


A system of forces act along the sides of a $\square$


$\square$ magnitude and direction. Find the point where the resultant cuts the OY axis. Take the horizontal and vertical components of the resultant as X $Y$ and the intersection point of $O Y$ axis and resultant as $P \equiv(0, y)$

## 281) $A B C D$ is a rectangle. A system of forces acts along $\overrightarrow{A B}, \overrightarrow{B C}, \overrightarrow{D C}, \overrightarrow{D A}, \overrightarrow{A C}$ <br> and $\overrightarrow{A E}, \overrightarrow{C D}, \overrightarrow{D A}$ The midpoint DC is E . The magnitude of the forces are $4 \sqrt{3}, 4,2 \sqrt{3}, 2,10 \sqrt{3}, 4 \sqrt{3}, 8,6 \sqrt{3}$ respectively. Find the magnitude, direction of the resultant force and the points where the resultant force cuts the $\mathrm{AB}, \mathrm{BC}$ sides. $\mathrm{AB}=4 \mathrm{~m}, \mathrm{BC}=3 \mathrm{~m}$. The system of forces is equivalent to two forces $\mathrm{P}, \mathrm{Q}$ along BC and AL . L is a point on CD . Find the positions of $\mathrm{P}, \mathrm{Q}$ and L

282) Forces $\underline{i}+3 \underline{j}$ and $3 i+3 \underline{j}$ acts on a rigid body. $i$ is the unit vector in the horizontal direction. Find the resultant vector along with its magnitude and direction.


$A B C D$ is a square of side 2 m . Forces of magnitude $2,3,4, P, Q, 4 \sqrt{2} \mathrm{~N}$ acts on the rigid body along the sides $\overrightarrow{A B}, \overrightarrow{B C}, \overrightarrow{C D}, \overrightarrow{D A}, \overrightarrow{A C}, \overrightarrow{B D}$. Find
283) ABCDEF is regular hexagon. Forces of magnitude $6 \mathrm{P}, 2 \mathrm{P}, \mathrm{P}, 7 \mathrm{P}, \mathrm{P}, 2 \mathrm{P}$ acting along the sides $\overrightarrow{A B}, \overrightarrow{B C}, \overrightarrow{C D}, \overrightarrow{D E}, \overrightarrow{E F}, \overrightarrow{F A}$ respectively. The length of a side is 2 am . Show that system is equivalent to a couple and find its magnitude.

$A B C$ is an equilateral triangle. The perpendicular distance from $A$ to the opposite side is $A D$. $A D C E$ is a rectangle. $A B=2 m$. Forces of magnitude $P$, $4,2,2,3 \sqrt{3}, 4,2$, Q Newton forces act along the sides $\overrightarrow{B A}, \overrightarrow{B D} \overrightarrow{D C}, \overrightarrow{A E}, \overrightarrow{E C}, \overrightarrow{D E}, \overrightarrow{C A}, \overrightarrow{D A}$. When the resultant force R acts
$\square$


| Point | Position Vector | Force |
| :--- | :--- | :--- |
| A | $2 \mathrm{i}+5 \mathrm{j}$ | $\mathrm{P}(\mathrm{i}+3 \mathrm{j})$ |
| B | 4 j | $-\mathrm{P}(2 \mathrm{i}+\mathrm{j})$ |
| C | $-\mathrm{i}+\mathrm{j}$ | $\mathrm{P}(\mathrm{i}-2 \mathrm{j})$ |


$\square$



letters. Three new forces $\mathrm{Q}, \mathrm{R}, \mathrm{S}$ newtons acting along the sides AF, FO, OA respectively, of the triangle AFO are added to the system. Find the values of $\mathrm{Q}, \mathrm{R}, \mathrm{S}$ in terms of P , in order that the combined system is. i) in equilibrium,
$\square$
(2007 A/L)


308) a) Define the dot product $\mathbf{a} . \mathbf{b}$ of two vectors $a$ and $\mathbf{b}$.


If the system is in equilibrium, find $\mathrm{L}, \mathrm{M}$ and N in terms of P . (2011 A/L)
309) $\underline{a}=\underline{i}+\sqrt{3} \underline{j}$ where $\underline{i}$ and $\underline{j}$ have the usual meaning $\underline{b}$ is a vector with magnitude $\sqrt{3}$. If the angle between the vectors $\underline{a}$ and $\underline{b}$ is $\frac{\pi}{3}$, find $\underline{b}$ in the form $x \underline{i}+y \dot{j}$ where $x(<0)$ and $y$ are constants to be determined.
(2012 A/L)


b) The coordinates of the Points $\mathrm{A}, \mathrm{B}$ and C with respect to a rectangular Cartesian axes Ox and Oy , are $(\sqrt{3}, 0)(0,-1)$ and $\left(\frac{2 \sqrt{3}}{3}, 1\right)$ respectively. |  |
| :---: |
|  |





that the line of action of the single force passes through the point $D\left(\frac{11}{3},-\frac{1}{3}\right)$.
(2015 A/L)

$\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{OQ}}=(1-\lambda) \overrightarrow{\mathrm{OD}}$, where $0<\lambda<1$. Show that $\overrightarrow{\mathrm{PC}}=2 \overrightarrow{\mathrm{CQ}}$.
(b) In parallelogram $A B C D$, let $A B=2 \mathrm{~m}$ and $A D=1 \mathrm{~m}$, and let $B \hat{A} D=\frac{\pi}{3}$

Also, let $E$ be the mid-point of $C D$. Forces of magnitudes $5,5,2,4$ and 3 newtons act along $\mathrm{AB}, \mathrm{BC}, \mathrm{DC}, \mathrm{DA}$ and BE respectively, in the directions indicated by order of the letters. Show that their resultant force is parallel to $\overrightarrow{A E}$, and find its magnitude.
Also, show that the line of action of the resultant force meets $A B$ produced at a distance $\frac{3}{2} m$ from B.
An additional force action through C is now added to the above system of force so that resultant force of the new system is along $\overrightarrow{A E}$. Find the magnitude and direction of the additional force.
(2016 A/L)
319)
$\alpha(>0)$ is a constant. Using scalar product. Show that AÔB $=\frac{\pi}{2}$

$\square$



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