STRAIGHT LINE

A/L Combined Maths

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Find the area of the triangle ABC where A (1,2), B(2, -3), C(-3, 1).

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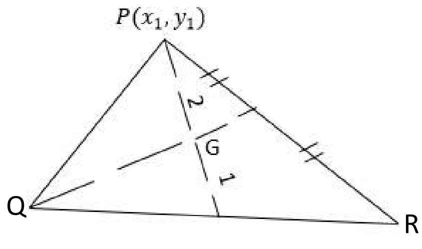
5. Show that the coordinates of the centroid G of a triangle of vertices $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$ is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$. Hence find the centroid of the triangle of vertices (1, 2), (-2, 3) and (5, 7).



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b) Find the coordinates of the point R that divides the line PQ at a ratio PR : RQ = -4 : 3 such that P(4, 2) and Q(-3, -1).

c) Find the coordinates of the point P that divides the line MN at a ratio MP : PN = -3 : 5 such that M(1, 1) and N(3, -2).

9. The coordinates of A, B, C vertices of a triangle ABC are (2, 1), (3, 2) and (-1, 2) respectively. P divides the line AB internally at a ratio AP : PB = 2 : 1. Q divides the line AC externally at a ratio AQ : QC = 5 : 1 and R divides the line BC internally at a ratio BR : RC = 3 : 4. Find the area of the triangle PQR.

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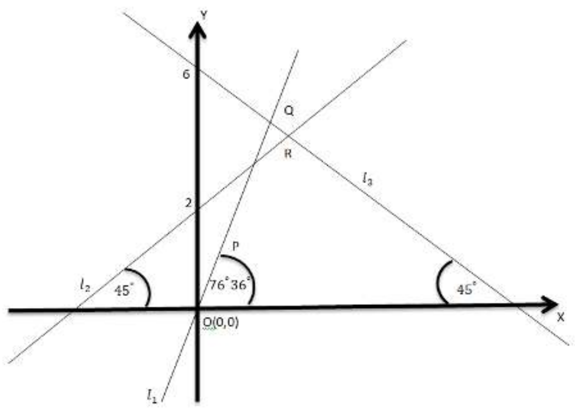

12. The equations of the lines PQ, QR and RP of the triangle PQR are $y = 2x - 1$, $y = 3x - 2$ and $y = -x + 6$. Find the centroid of the triangle PQR.

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17. Find the equations of the straight lines which joins the pairs of points given below.
   a) P (1, 1) and Q (3, 4)
   b) P (1, -1) and Q (-3, 4)
   c) $(x_0, y_0) \equiv (1, 1)$ and $(x_1, y_1) \equiv (3, 4)$

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28. Show that the straight-line having intercepts a and b on OX, OY axes respectively can be written as $\frac{x}{a}+\frac{y}{b}=1$. Taking $b>a>0$, show that the area of the triangle which is made by the lines $\frac{x}{a}+\frac{y}{b}=1$, $\frac{x}{b}+\frac{y}{a}=1$ and the OY axis is $\frac{1}{2}\frac{ab\,(b-a)}{(b+a)}$. Hence find the area between $\frac{x}{3}+\frac{y}{2}=1, \frac{x}{2}+\frac{y}{3}=1$ and OX axis.

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38. ABCD is a rectangle on the XOY plane. AB line is on the $y = 2x + 1$. Coordinates of C are $C \equiv (3, 1)$ and the point E(1, 4) is on the line DA. Find the acute angle between the lines BE and BD.

39. Find the equation of the straight line which passes through the intersection point of the two equations $x + 3y - 2 = 0$ and $2x - y + 4 = 0$, and passes through the point (-1, -1), without finding any coordinates.

40. Find the equation of the straight line which passes through the intersection point of the two equations $x + 3y - 2 = 0$ and $2x - y + 4 = 0$, and having an intercept of -6, without finding the coordinates of the intersection point.

41. without finding the coordinates of the intersection point, find the equation of the straight line which passes through the intersection point of the straight lines $3x + y - 2 = 0$, $2x - 3y + 1 = 0$ and parallel to the straight line $y = \frac{1}{7}x - 2$.

42. without finding the coordinates of the intersection point, find the equation of the straight line which passes through the intersection point of the lines $x + 3y - 2 = 0$, $2x - y + 4 = 0$ and perpendicular to the line $y = \frac{2}{3}x + 4$.

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47. Find whether the point $P(\frac{1}{2}, 5)$ lies inside the triangle ABC of the sides AB, BC, CA represented by the lines $2x - 3y - 1 = 0$, $2x + y - 5 = 0$, and $5x - y + 4 = 0$ respectively.

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50. The equation of the diagonal BD of the square ABCD is $2x - y + 4 = 0$. The coordinates of the vertex A are (2, 3). Find the equations of the remaining diagonal and the remaining sides of the square.

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55. Find the acute angle bisector of the straight lines $l_1 = 3x + 4y - 1 = 0$ and $l_2 = 5y - 2 = 0$.

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58. If the length of the perpendicular drawn from any point on the line $l_1 = x + 9y + 3 = 0$ to the line $l_2 = x - 3y + 1 = 0$ is two times the length of the perpendicular drawn from any point on l_1 to the line $l_3 = 2x + 6y + \lambda = 0$, find the value of λ.

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77. Let $A \equiv (4,3), B \equiv (10,7)$ and $C \equiv (0,9)$. Let the midpoints of the sides BC, CA and AB of the triangle ABC be D, E and F respectively.

    a) Find D, E and F.

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91. Let ABC be a triangle where $A \equiv (3, -1), B \equiv (3, 4)$ and $C \equiv (-3, 3)$.
    a) Find the equations of the lines AB, BC and CA.
    b) Find the equations of the medians AD, BE and CF.
    c) Using another method, verify the answer obtained for the coordinates of the centre.

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97. Find the intersecting points of the curve $y^2 = x^2 - 8$ and the line $y = x - 2$. Find the distance between those points, A and B.

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102. Find the coordinates of the image point of (α, β) through the line $lx + my + n = 0$. The two lines going through the origin which makes equal angles with the line $x - y = 0$, intersect the line $x = 2$ at A and B. If the image of the midpoint of the line AB formed from the line $2x - y + 1 = 0$ lie on the y axis, find the equations of the two lines.

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110. In the triangle ABC, equations of the sides AB, BC, CA are $x - y + 1 = 0$, $x + 7y - 1 = 0$ and $x + y + 3 = 0$ respectively.
   a) Find the coordinates of the points A, B and C.
   b) Find the internal angle bisector of A.

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115. Let ABC be a triangle where $A \equiv (4, -2), B \equiv (-2, 4)$ and $C \equiv (5, 5)$.
   a) Find the equation of the sides AB, BC and CA.
   b) Find the bisectors of the external and internal angles of $B\hat{A}C$ and $A\hat{B}C$.
   c) Find the intersecting point of the internal angles bisectors and obtain the centre of the in-circle.
   d) Obtain the centre of the external circle touching the line AB.

116. Let $x - 2y = 0, 2x - y = 0$ and $x + y = 1$ be the equation of the sides of a triangle. Show that the triangle is an isosceles triangle. Find the area of the triangle and the coordinates of the circumcentre.

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121. Show that the points (x_1, y_1) and (x_2, y_2) lie on the same side or opposite sides of the line $ax + by + c = 0$ if and only if sign of $(ax_1 + by_1 + c)$ and [redacted]

122. Let ABCD be a rhombus. Let A and C be on the vertex $x + y = 0$, and B and D are on the vertices $x - y = 0$ and $5x - y + 9 = 0$ respectively. Let the side AB be parallel to the line $x - 2y = 0$. And the side BC be parallel to the line $x - 3y = 0$. Find the equation of the sides of a parallelogram.

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127. Find the equations of the diagonals of the parallelogram formed from the straight lines $ax + by + c = 0, ax + by + d = 0, a'x + b'y + c' = 0$ and $a'x + b'y + d' = 0$.

a) If $(a^2 + b^2)\,(c^1 - d^1)^2 = \left(a'^2 + b'^2\right)(c - d)^2$, show that the parallelogram is also a rhombus.

b) Show that the area of the parallelogram is $\left|\frac{(c - d)(c' - d')}{ab^1 - a^1 b}\right|$.

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139.

a) Find the coordinates of the point C which internally divides the line joining the points $A \equiv (3,1)$ and $B \equiv (-3,-5)$ to the ratio 2:3.

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166. Let ABCD be a rectangle where the coordinates of A and C are (2, 3) and (9, 4) respectively. If the diagonal l perpendicular to the line $x + y = 0$, find the equations of the sides of the rectangle. AECF is a rhombus. Its area is 5 times the area of the rectangle ABCD. Show that the length of the diagonal EF is $15\sqrt{2}$, and then find the equations of the lines going through E and F and parallel to AC.

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175. Show that the line $(1+k)x - (1-2k)y + 5 - k = 0$ represented by l, goes through the fixed-point O, when k is a constant. If the equation perpendicular to l is x - y+1= 0, find the equation of l. Find the two points P and Q on l which are $\sqrt{10}$ units away from O.

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181. If the straight-line $y - y_0 = $ m' $(x$ - $x_0)$ makes an angle of $y - y_0 = $ m $(x - x_0)$, (m≠1) with the straight line 45^0, prove that $m' = \frac{1+m}{1-m}$ or $m' = \frac{-(1-m)}{1+m}$. Let one vertex of a parallelogram be (-1, 1). The centre of the parallelogram is (1, 5). Find the other three vertices of the parallelogram.

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similar area. Prove that P has two positions, and one can be named as P_0, and that PP_0 AB is a rectangle.

Find the coordinates of the fourth vertex Q, such that P_0ABQ is a rectangle.

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187. The perpendiculars drawn from A (-8, 10), B (1, 2) and C (1, 11) to the sides B'C', C'A' and A'B' of the triangle A'B'C' are concurrent. The lines B'C', C'A and A'B' lie on the lines $3x - y - 5 = 0$, $x - 2y = 0$ and $x + \lambda y - 15 = 0$ where λ is a constant. Find the value of λ. Also prove that the perpendiculars drawn from A',B',C' to BC, CA and AB respectively are concurrent.

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205. The vertices B and C of a triangle ABC lie on the line $4x - 3y = 0$ and the x-axis respectively. The side BC passes through $\left(\frac{2}{3}, \frac{2}{3}\right)$ and has slope m.
    a) Find the coordinates of B and C in terms of m.
    b) Show that $OB = \left|\frac{10(m-1)}{3(3m-4)}\right|$ and $OC = \left|\frac{2(m-1)}{3m}\right|$, Where O is the origin.
    c) If ABOC is a rhombus, find the two possible values of m, and the corresponding coordinates of A.

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b) The equation of a straight line through a point (x_0, y_0), is given in the parametric form $\frac{x - x_0}{a} = \frac{y - y_0}{b} = t$, where $a^2 + b^2 = 1$ and t is a parameter. Show that $|t|$ is the distance from the point (x_0, y_0) to the point (x, y) measured along the line.

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